# THE GEOMETRY

OF ALL

EQUATIONS, CUBIC, and BIQUADRATIC, by a *Circle*, and any one only *Parabole*.

# *Clavis Geometrica Catholica:*

## SIVE

# JANUA ÆQUATIONUM

## *RESERATA:*

Methodus omnes Æquationes quomodolibet affectas, quartum gradum non Excedentes; nempe,

*LINEARES,*
*QUADRATICAS,*
*CUBICAS,*
*BIQUADRATICAS*
} Construendi;

Ipsarumque omnes radices, tam falsas quàm veras eliciendi; absque ope *Mesolabii*, *Anguli Trisectionis*; absque Æquationum *Reductione*, *Depressione*, vel quavis aliâ præparatione, per Circulum, & (quamlibet) unicam Parabolam.

*Idque*,

Ex unâ unicâ quidem Regulâ Generali; quâ simplicior, perfectior, generalior, intellectu facilior, praxi accomodatior, non est excogitanda, vel exoptanda.

Demonstrationibus munita, Figuris ad quamlibet Æquationem insignita, & Æquationibus numeralibus, pro varietate casuum, ad quamlibet Figuram adaptatis Exemplificata.

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*In usum Tyronum; opus adhuc desideratum.*

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Ἐὰν ᾖς φιλομαθὴς, ἔσῃ πολυμαθής.

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A *THO.* BAKER.

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*LONDINI*, Typis *J. Playford*, & prostant venales apud *R. Clavel*, ad Insigne *Pavonis* in Cœmeterio D. *Pauli*: M. DC. LXXX. IV.

# THE
# Geometrical Key:
## OR THE
# GATE of EQUATIONS
## *UNLOCK'D*:

A New discovery of the Construction of all Equations, howsoever affected, not exceeding the fourth Degree; *viz.* of,

*LINEARS,*
*QUADRATICS,*
*CUBICS,*
*BIQUADRATICS;*

And the finding of all their Roots, as well false, as true; without the use of *Mesolabe, Trisection of Angles*; without *Reduction, Depression*, or any other previous preparation of Equations, by a Circle, and any (and that but one only) Parabole.

*And this*,

By one only General Rule; than which a more simple, more perfect, more general, more easie to be understood, or more fit for practice, cannot be devised or wished for.

Fortified with Demonstrations, Illustrated with Figures, to each Equation; and Exemplified with numeral Equations, (according to all the varieties of cases,) adapted to each Figure.

*For the use of Young Mathematicians, a Work hitherto desired.*

Ἐὰν ᾖς φιλομαθής, ἔσῃ πολυμαθής.

By *THO. BAKER.*

*LONDON*, Printed by *J. Playford*, for *R. Clavel*, at the *Peacock* in S. *Paul's* Church-yard: M. DC. LXXX. IV.

*Inclytis ac præstantissimis viris, nobilissimis*

MATHEMATICIS,

*Reverendissimo in Christo P. ac D.*

D. SETHO

*De* Sarum *Episcopo, primitias;*

Nobilissimo ac Honorando

*FRANCISCO ROBARTES*, Armig.

Tam Generis Nobilitate, quam Virtutum splendore ornatissimo, Filio

*Honorandissimi D. D.*

JOHANNIS

Comitis de *Radnor*, &c.

*Spectatissimo & admodùm Colendo*

D.D. *JOSEPHO WILLIAMSONO*,

Societatis Regalis Præsidi vigilantissimo,

In Testimonium Gratitudinis & Observantiæ, Lucubrationes hasce quales quales sunt humiliter devovet

*T. B.*

TO THE MOST

*Renowned and Excellent Men,*

*MOST FAMOUS*

# MATHEMATICIANS:

*The Right Reverend Father in God,*

## SETH,

Lord Bishop of *Sarum*, these his first Fruits:

*The most Noble and Honourable,*

## FRANCIS ROBARTES, *Esq;*

A Zealous Fautor of Learning;

*Son of the Right Honorable,*

## JOHN,

EARL of *Radnor*, &c.

The Right Worshipful,

## Sr *JOSEPH WILLIAMSON*, Kt

And President of the Royal Society, in Testimonial of his Gratitude and Observance,

*These his Mathematical Lucubrations (such as they are) most humbly devotes,*

T. B.

*TO THE YOUNG*

# MATHEMATICAL READER

## PARANETICΩS.

THERE *is none who is not fervently desirous of knowledge; whom the love of truth doth not vehemently inflame, and set on fire: Now this enflamed desire no Science gives such satisfaction to, as* Mathesis *(the Princess of all Sciences) doth: Whose misfortune, yet whether it be more to be wondred at, or pitied, I cannot determine: For tho she deservedly challenges this privilege as peculiar to her self, to caress and affect her Votaries (upon the most clear and evident intuition of Truths) with more incredible and satisfactory pleasure, than other Arts or Sciences can or dare pretend to, and with such certainty, as leaves in mens minds, as little room as power to doubt; yet, of late years, to gain but very few Devoto's to her Shrine. The most certain skill of the most expert Physician is founded, but upon uncertain speculations, and his second conceptions of the Symptoms of a disease justle by his first Sentiments: Now* Galen's *Authority, anon* Paracelsus's *prevails; both by turns commended, condemned: And he will suppose himself to be no mean Artist, if from some hundreds of Conclusions, he can but fancy some of them more likely to be truer than their opposites; and can but hope that he is able easily to manage others of them against opposers; and oft-times hath none, on which he may rely, as exempted from all manner of Scruple.* The Old Aristotelian

*Lectori benevolo tyroni*

# MATHEMATICO

## PARANETICΩS.

NEMO prorsùs est, qui scire non unicè expetat; quem veri amor non vehementèr inflammet; huic verò tam inflammato desiderio una propemodum *Mathesis* (omnium artium facilè princeps,) satisfacit; Cujus tamen infortunium an demirandum vel miserandum magis sit, planè ignoro: Quamvis enim quasi propriam hanc sibi palmam vendicet, ut Cultores suos mirum in modum afficiat & demulceat, (idque incredibili illâ voluptate, quæ ex claro & evidente veritatis intuitu exurgit, ut nullus omninò dubitandi locus superesse possit,) paucos tamen hujusce deliciis captos, perpauciores nunc dierum unicè devotos conciliaverit. Certissima optimi medici peritia speculationibus incertis nititur, primique ferè morbi sypmtomatum & causarum conceptus secundis, & secundi tertiis cedunt. Apud quem nunc *Galeni*, nunc *Paracelsi* valet Authoritas; & uterque per vices excellit, vilescit; cui hæreat, omninò subdubitat. Præclarè secum actum arbitrabitur, si ex aliquot Conclusionum Centuriis, alias putet vero-similiores oppositis, alias speret tueri se haud difficultèr adversus oppugnantes posse; & sæpè habeat nullas, quibus secluso omni dubitationis scrupulo acquiescat. Explosis ferè veteribus

*Aristote-*

*licorum* principiis, nova *Carteſiana* (ne dicam rationi magis congrua) ſubindè ſubeunt; ipſorumque Carteſianorum nonulla à *Boylianis* recentioribus (experientià comprobatis) peſſundantur. Hujuſce autem Triumviratûs quilibet, in rerum naturalium cauſis (quibus non omniò poſſit ſubeſſe falſum) aſſignandis, non poſſe toto cælo ſæpiſſimè errare, nullus dubito; quippe qui nullo Topico, (unde vel maximè probabilia eliciantur Argumenta,) ſuffigere poſſint, τὸ quod erat demonſtrandum. At horum aſſeclis ſcholæ ubique adeò arctantur, ut vix inibi reſpirantibus aer ſuppetit: Unica verò Mathematica (ubi non una aliqua expanditur veritas, ſed planè innumerabiles; eæque non vulgares & obviæ, ſed nominis ſublimioris eximiæ plerunque atque reconditæ, maximéque admirabiles perſpicuè demonſtrantur) vacua, araneoſa & contempta jacet. Ab hâc quidem, tanquam à loco peſtiſero (nunc dierum) refugitum; ad illas verò, tanquam ad oraculum Delphicum (vel, ut ad Candida Tecta Columbæ,) turmatìm confugitum eſt.

Reges olim Principeſque hujus amænitate & ſimplicitate adeò ſunt allecti, ut (cæteris regnorum ſuorum deliciis ſuas ſibi res habere juſſis,) in Clientelam ſe ſupplices contulerunt, & nomina in Militiam ſolam Mathematicam dederunt, tanto pretio hanc ſcientiam redimentes. Si *Anacharſim* Schytam, & *Heraclitum* Epheſium commemorem, qui regna hæreditaria Contemplationi hujus poſtpoſuerunt; iiſdemque relictis, ad Philoſophorum ſedere pedes, quàm regiis inſidere ſoliis maluerunt: Si *Atlantem Mauritaniæ* Regem, quem (ob artem, quam inſigniter calluit, Aſtronomicam) humeris cælos ſuffulcientem, fabuloſa nobis exhibet Antiquitas: Si *Agathoclem Siciliæ* Regem, *Ptolomeum Philadelphum*, *Alphonſum*

lian *Philosophers Principles are exploded by the fresh start of the new (I say not, more rational)* Cartesian; *and some of the* Cartesian *again baffled by the newer and truer experimental* Boylian: *And each of them, as far to seek of the true unerring Causes of natural things, as they are of the reason of the Magnetic vertue of the Load-stone; in asmuch to any Topic (from which they draw their most concluding Arguments) they cannot subfix a* quod erat demonstrandum. *And yet their Schools are so stuffed with Proselytes, that they have scarce room to breath in: But the Mathematic (School) only, (in which, not some one Truth only is expanded, but even innumerable; and those, not mean and obvious, but most high, admirable and mysterious are cleerly demonstrated) lies orbate and neglected. From this they fly, as from some Pest-house; but to those, they troop, as to a Delphic Oracle, or as Doves to white Dove-houses. Kings and Princes heretofore, have been so enamored with her simplicity and pleasantness, that (forsaking all the delights of their Kingdoms) have made their addresses to her Shrines, paid Homage to her Altars; thus redeeming science, at so great a price. Should I mention* Anacharsis *the Scythian, and* Heraclitus *the Ephesian, who prefer'd the Contemplation of Philosophy before their hereditary Kingdoms, and chose rather (leaving those) to sit at the feet of Philosophers, than on their Kingly Thrones: Should I recount* Atlas *King of* Mauritania, *whom (for his Astronomic skill, wherein he excelled) Antiquity hath fabled to bear up the Heavens on his shoulders; or* Agathocles *King of* Sicily, Ptolomy *of* Philadelphia, Alphonsus, *of* Castile,

*phonsum Castellæ*, *Fredericum Daniæ*, *Gulielmum Lantgravium Hassiæ*, &c. Imo verò, si Imperatores, *Cæsarem* sc. *Adrianum*, *Theodosium*, &c. qui (studiis hisce, Imperatoribus quidem dignis, incumbentes) scriptis suis, quam bello gestis (quamplurimis licet & inclytis) insigniores evasêre; non tantum opprobrium, sed & pudorem ævo huic degeneri tacitè injicerem. Si quosdam alios ordinis inferioris reputem, hujusce gratissimis (pæne dixeram divinis) fascinationibus mirum in modum extra se raptos: * Si *Archimedem*, spretâ Cuticulæ curâ, apud balneam cinere focario Figuras Geometricas exarantem, digito Lineas ducentem; thermisque etiam nudum exilientem (cùm Aurifabri furtum deprehenderat) suumque εὕρηκα ingeminantem: Si *Pythagoram*, (investigatâ Trianguli Rectanguli proprietate) Hecatomben Musis immolantem: Vix illos æquè debito honore prosequerer, ac nobismet ipsis acerrimè sc. perstringendo) ruborem non immeritò incuterem. Denique quamvis intrinseca ejus forma & pulchritudo nonnullos etiam infimi subselii homunciones, qui à limine solum salutarunt Arcana sua neutiquam lustrantes) eò adegerit, ut demirarentur; perpaucos certè videre est, aut illam penitiùs introspexisse & calluisse, nedum Cultores suos remunerasse. Quâ de causâ gratiam, existimationem & authoritatem apud Uulgus indies amitteret, formosa hæc Dea, hariolari nequeam. Utrum eò quod cùm scientia liberalis (sive ingenua) fuerit, ideoque tenacissimis illis Lucronibus, (in hominum numero vix censcendis) incongrua, qui (ad rem plus satis attenti) marsupia quàm animos locupletare malint; aut ideò, quòd spem omnem ad ἀκμὴν perveniendi, (quibus summis in votis est, aut Cæsares, aut nullos esse) abjiciunt;

* *Plutarch in vitâ Marcelli.* p. 307.

Frederic *of* Denmark, William Lantgrave *of* Hassia, *&c. Yea, but should I mention Emperors*, viz. Cæsar, Adrian, Theodosius, *&c. who (devoting themselves wholly to these Studies, worthy indeed of Emperors) rendred themselves more Illustrious, by their Writings, than by their Warlike (tho many and great) Atchievements; I should but silently shame and reproach this our degenerate Age. Nay, should I but mention, how strangely the minds of some others of a lower Sphere have been Captivated with its (I had almost said, divine) charming delights: How it forced* * Archimedes, *sometimes to forget his repast, and the care of his body, and at the bath on the heath to exarate* Geometrical *Figures, and to draw Lines with his Finger; nay, (upon the detection of the* Gold-*smiths theft) to leap naked out of his Bath, and to ingeminate his* ἕυρηκα: *How* Pythagoras *(upon the discovery of the propriety of a Rectangle Triangle) to immolate an Hecatomb to the Muses*, &c. *I should not more loudly honour them, then tacitly check and upbraid our selves. Lastly, tho her intrinsic worth and beauty hath compelled others of the lowest Orbe, (who (saluting her only at the threshold) never entred, or had the least glimps of her Arcana's or inner Rooms) to admire her; yet certain it is, very few are skilled in her mysteries; by which means it comes to pass, that she is as little regarded, as her Clients rewarded. For what cause this beautiful* Goddess, *should thus suffer an Eclipse in her glory and esteem with the Vulgar, now a days, I cannot divine; Whether it be, she being a liberal Science, and therefore (on that account) unsuitable to the humours of those close-fisted Misers, (who are scarce to be reckoned among the number of men) who love to have their purses enriched rather than their minds: Or, whe-*

*Plutarch in the life of Marcellus. p. 307.

abjiciunt; Aut eò quòd ardua nimis & ſpinoſa artem hanc præ ſe ferre (quod faciliùs autumem) exiſtimant: Aut quæcunque tandem illa fuerint μορμολύκεια, quæ recentes poſtquam vel primori linguâ Cupedias haſce libaverint, abſterrerint, (cum demum eorundem ſedulitatem & ſolertiam egregio Evidentiæ & Certitudinis præmio abundè compenſaſſet) indicare, nedum dijudicare non eſt meum. At verò hoc (Amice Lector) obſervanti expertoque mihi ſæpiuſculè occurrit; quòd cùm Tyrones nonnulli (non contemnendo tam operæ quam olei diſpendio) problema etiam ad Carceres uſque, juxta Artis Analyticæ Regulas inſequuti ſunt, videlicet ad Æquationem ad gradum elatiorem aſcendentem, quàm præſentiſcebant, ſc. ad tertiam, quartam, altioremve poteſtatem, (cujus reſolutio quidem Arithmetica ſubdifficilem, conſtructio verò Geometrica (quantum ſcivere) impoſſibilem eſſe, utpote toti orbi Mathematico peregrinum adhuc & ignotum) actum eſt derepentè de Matheſi, & quâ Problemati quà Arti valedixeruut. At hoc quidem (Lector) utrùm demirandum magis, an abſurdum meritò dubitandum; nempè, quod illud ipſum quod tibi animos adderet, animos adimeret, quod potiùs ut ulteriùs concitatiore impetu involando incitaret, te percellerit: Vere ardua (multo minus illa, de quibus præjudicare pro more noſtro plus ſatis ſolemus) non tantum ingenii aciem non retuderint, verum etiam virtutis animique tibi coti fuerint, conatuſque animaverint: Imò verò eo ipſo nomine, quod admiranda ſunt & difficilia (modò poſſibilia) ſpem animumque adderent, aciemque tuam exacuerint. Quò ſæpiùs humi abs *Hercule* proſtratus *Antæus*, eo vivacior exurgit, & fortior contendit *. Tam amiſſam gloriam reparandi ſpes,

* *Hom.* Il. pag. 704.

*ther their despondency of ever arriving to any considerable eminency of height, (it being as good, to be nothing, as not a None-such, or, but a Spy, to an Art :) or whether it be the fancied difficulty and knottiness of the Study it self, (which I have most cause to suspect.) Or, what that supposed Mormo may be, that forestals and prejudiceth some newly entred, & scares others, who have tasted some of her sweets from farther Essayes (which in fine, would have crowned their sedulity and diligence, with evidence and certainty, (both which this Art carries, and no other doth, and which is reward enough to compensate their pains) I shall not undertake to determine. Only this one thing (Reader) in my little experience, hath occurred to my observation oft, that when some Tyro's (with as great expence of pains, as of time) have according to the Rules of the Analytic Art, pursued a Problem to its end,* viz. *to an Equation ascending to an higer degree,* (viz. *to the* 3, 4, *or higher Power) than they expected, whose Resolution Arithmetical they conceived very difficult; but* Geometrical Construction *impossible, (as being yet unknown to the Mathematic world; they have bid farewel, as to the Problem, so to the Art it self too. But this (Reader) is as much absurd, as strange :* viz. *That what should recommend this study to thy reason should discourage thee; that what should animate thy diligence, and quicken thee to a further Essay, should decrest and dispirit thee. Real difficulties (much less conceived prejudices) should be so far from blunting thy edge, that they should rather be the whetstone of vertue and sharpen thy endeavours : Why may not the same things, which (for the excellency of them) are the objects of thy admiration, be (for their possibility) as well the object of thy hope, and the Encouragement of thy industry?* Antæus *recovered more strength by each fall* Hercules *gave him.* * *The possible hopes of Redeeming or retrieving their lost*

*Hom. *Il.* pag. 704.

ſpes, *Græcos*; quam compluſcularum victoriarum recordatio *Trojanos* ad prælia accendit. Non adeò ardua hujus artis pericula, & inexpugnabilia, quin (uti bello) aut Solertia aut Caſu, aut utroque ſunt ſuperabilia. Præclara *Alexandri* geſta *Cæſar* recolens, propriamque exinde exprobrans ignaviam, ad auſa inclyta auſpicanda novos ſumpſit animos, proſperéque pugnatum eſt. *Appelles* ſummâ incuriâ penecillum ſuum in Tabellam iratus projiciens, canis rabidi ſalivam (quam malè depinxerat, fortuitò ſed graphice correxit, & felici hoc infortunio, detur verbo venia) quàm arte clarior evaſit. Ludit in omnibus (etiam in ſtudiis humanis) fortuna Dea (verius dixerim divina providentia;) & qui non Artis, Aleæ ſæpiſſimè fit magiſter: Et quemadmodum plurimum laboribus & vigiliis, ita Caſui aliquando nonnihil tribuendum; qui tibi ſtudiis hiſce invigilanti nova aliquando & præter opinionem ſuggerat. Sagaces ſedulique Lapidis Philoſophici indagatores quamvis aurem propoſitum nequaquam attingerint, attamen non omnem plerumque ludunt operam; dum fortuitò in aliud quod tam laboribus quam inpenſis non indignum incidunt. Haud aliter mihi evenit in Regulæ hujus univerſalis inventione (de quâ hoc Tractatu agimus) nempè de Conſtruendis Æquationibus omnibus quartum gradum non excedentibus, & de determinandis illarum locis: quibus plurimi non vulgaris eruditionis Geometrici diu inſudarunt, multumque lucubrationibus ſuis oleum, exitu non ex æquo fælici, inſumpſerunt.

Non eò quod (fateor) mihi cor limante Minervâ,
Acriùs, & tenues finxerunt pectus *Athenæ*;
Sed quòd (dexterior ſolito) mihi dexter *Apollo*
Adfuit ——

Dum

*lost honour enflamed the* Græcians, *as well as their being flesht with frequent Victories did the* Trojans, *to fight. The difficulties of this Art are not so insuperable, (but as in War) they may be overcome, either by industry or fortune, or both.* Cæsar *upbraided his own slackness with the memory of* Alexander's *conquests, even upon the bare sight of his Picture only, and enspirited himself to high bold and daring attempts, and proved successful.* Appelles *in anger carelesly throwing his Pencil, accidentally (tho inartificially) well shaped the ill drawn Vomit of his Painted Dog; and this prosperous mischance made him more famous, than his Art could doe. Fortune (or Providence rather) sports it self (pardon the word) in all things, (even in humane studies) and he that is not Master of Art, may yet be so of a Chance. As (Reader) thou maist attribute much to thy sedulity and industry; so somewhat sometimes to fortune too, which may (if industrious in thy inquest) favour thee perhaps with some new invention beyond thy expectation. Those busie enquirers and searchers after the Philosophers Stone tho they lose their aim, yet not usually all their labour; but stumble oft on something worthy their diligence and expence. It chanced to me thus in the invention of this universal Rule, of which we now treat; namely, of the Construction of all Equations not exceeding the fourth degree, and of determining their places; about which many Learned* Geometricians *have sweated and spent much Oyl at their* Lucubratory *Tables, but perhaps not with equal success. Not, that I had quicker brains, but better luck. For whiles busying my self (who pretend not to Learning, nor to the Profession of the Mathematic Art, but one, who at some subcisive hours, for diversions sake, its study much delights) in an Analytic inquest (by way of Porisma) of what* des Cartes *had*

*Written*

Dum enim Analyticam disquisitionem illorum quæ *Cl. Cartesius* de eodem Argumento * Porismaticos meditarer, (qui me Mathematicum non profiteor, sed quem (si quando vacat) horis succisivis delectant Mathematices studia) fortuna (ars nequaquam) conatibus favens nostris, in methodum quæ Regulam ejus nimis strictam, ampliorem, imo (quod & ipse demiratus sum) facillime eam universalem redderet, præter institutum incidi. Quod dubiò procul Cl. illo & sagacissimo viro perspectum fuisset, si fors fausta illum (prout me) eò duxerat, ut Circulum à quovis Puncto in positione dato, per verticem cujusvis diametri in Parabolâ ductum (prout à vertice Axis fecit) descripsisset; & proprietatem quandam Parabolæ (huic instituto apprimè aptam natam) insuper animadvertisset. At *Bernardus* non videt omnia.

* Geom. l. 3. pag. 85.

Ad hoc tamen tam *Cl. Cartesius*, quàm alii eruditissimi Mathematici quà Antiqui quà Neoterici collimarunt: Horum verò speculationes eò tantùm vergebant, ut Mesolabum & Angulorum Trisectionem, si fors dederit, tandem demùm investigarint.

*Vieta* (primus Analysis speciosæ Inventor) hanc rem non perperam videtur ventilasse; at post accuratissimam disquisitionem non potuisse expedire & effectam dare cuilibet innotescat Tractatus ejus de Pseudo-mesolabo & supplemento Geometriæ consulenti.

Cl. *Outhedus* (nostras) postquam Æquationes nonnullas Cubicas prælibaverat * (quâ etiam solertiâ (uti inquit) alias innumeras Analytices studiosus poterit comminisci) sperat fore, ut illarum ope Mesolabum hactenus tenebris obvolutum in lucem tandem proferatur.

* *Cl. Math.* Chap. 18.

Quod

*Written on that Subject; Fortune (not Art) was pleased favourably to aspect my weak Endeavours; and when not designing it, I happened to hit on such an improvement of his Rule, as (to my great admiration) would render it universal: which (no doubt) that quick Lyncean-eyed man might have seen, and would have made a præ-discovery of, had it been his hap (as it was mine) to have described a Circle, from any Point in Position given, passing through the Vertex of any Diameter in a Parabole, as he did from the Vertex of its Axe; and withall had taken into consideration a certain propriety (than which none could have so well suited his design) belonging to the Diameter of any Parabole. But* Bernardus non videt omnia.

*And as this (no doubt) was famous* Des Cartes's *chief aim; so hath the Enquiry after this one thing especially, been the Subject of the chiefest Study of the choicest and learnedest Mathematicians of the former and latter Ages; but taking different Measures, it proved not equally successful. For their Speculations bended mostly to the invention (if possible) of the Mesolabe and Trisection of Angles.*

Vieta *(the fist Inventor of Specious Analysis) seems, not perfunctorily to have examined this matter; and after his most exquisite search, tacitly insinuates his own ignorance, as may be seen in his two Tracts; the one de Pseudo-mesalabo; the other, de supplemento* Geometriæ.

*Our Famous Mr.* Oughtred * *having prelibated some few Cubic Equations, hopes by their help, the Mesolabe hitherto involved in darkness, may at length be brought to light.* * *Cl. Math.* Chap. 18.

Quod etiam *Cartesius* ipse (unà cum Celeberrimis ejus Commentatoribus, *Francisco* à *Schooten*, *Huddenio*, *Florimondo de Beaun*, *Johanne de Wit*, (in Elementis Curvarum) (qui omnes Cartesii vestigiis pressissimè insisterunt;) & alii nonnulli (ut *Cl. Fermatius*, *M. de la Hire*, *Slusius* (qui cæteris palmam hâc in re præripere videtur) Conicarum Sectionum ope & Circuli aggressi sunt: Quorum quidem omnium hâc de re Lucubrationes neutiquam sunt contemnendæ; & uti quidem multis nominibus, ita eo præcipuè, quòd ad plana omnia & solida loca reperienda (quorum beneficio Æquationes ad elatiores gradus ascendentes possint construi) viam præstravere.

Notandum verò Clarissimum illum (*Cartesium* nempè) non ulteriores (quantum video) progressus fecisse, quàm istiusmodi Æquationes construendo, quibus supponendùm est (ut plurimùm saltem) secundum terminum deesse. Quem quidem (eâ de causâ opinor) nimis acrius perstrinxisse videtur *Bartholinus*, insimulando Cl. illum varia in hâc materiâ imperfecta reliquisse,*) quæ (ut ipse innuit) ad perfectionem deducere conatus est vir eximius *Florimundus de Beaune*, sed in medio cursu subsistens (morte præventus immaturâ) ultimam Authoris manum non passa sunt: At effecta dedit, & numeris omnibus absoluta nobis exhibuit idem *Bartholinus* (uti inquit) in duobus Tractatibus, de limitibus Æquationum, & dioristicâ methodo: perperam (ni fallor) asserens, necessitatem priùs resolvendæ, immò & resolutæ priùs determinandæ & definiendæ cujusvis Æquationis à Dioristice primò inventæ, quam ejusdem constructionem quilibet ausus sit aggredi.

* Barth. *Selecta Geom. Ep.* pag. 7.

Ob-

Des Cartes *himself, with those famous Commentators on him* Franciscus à Schooten, Hudden, Florimond de Beaun, John de Wit *(in his Elements of Curves) (all most pressingly insisting in his steps) and divers others (as* Fermat M. de la Hire, *and* Slusius *(who seems to outdoe them all) have attempted it, by the help of Conical Sections and a Circle; all whose pains therein, being of singular use, are not to be despised; as in many other respects, so in this especially; in that they find out all Plane and Solid places for the Construction of all Equations of higher Degrees.*

*But Note, it reaches no further (for ought I can perceive) than to such Equations, where the second term mostly must be supposed to be wanting. Upon which account, I suppose* Bartholinus * *is pleased to censure* Des Cartes, *that he had left many things, about this matter imperfect; which (as he insinuates)* Florimond de Beaun *afterwards endeavoured to bring to perfection, but being prevented by an immature death, desisted in the midway: Which again after that* Bartholinus *himself (as he says) hath perfected in his two Tracts, the one, of the limits of Equations; the other, in his Dioristics: Strangely (if I mistake not) concluding a necessity of every Equation to be resolved; yea, and being resolved of limiting and determining every of them, by a Dioristic first found, before ever any man may dare to attempt its Construction.*

* Selecta Geometr. Ep. pag. 7.

Ob defectum cujus, magnum (inquit) animadverto in Arte hiatum, cui replendo, non mediocre sufficit ingenium, quem ipse (acumine quo pollet non vulgari) accuratissimè postmodùm implevit. Quæ quidem an tam eximio usui fuerint in Æquationum radicibus Arithmeticè eliciendis non edisseram: At verò an tanti momenti, an tam prorsùs necessaria ad Æquationes Geometricè construendas (ut arbitratur ille, & contendit) doctorum arbitrio, postquam Tractatum hunc percurrerint, relinquendum esse censeo. In quo nullus dubito, quin reperiant, omnes Æquationes quomodolibet affectas quartum gradum non excedentes, Dioristice nequaquam obstetricante, ad amussim Geometricè construi posse; immo quidem vel absque ope Mesolabi, Sectionis Angulorum, sive aberit sive aderit secundus (sive quivis alius) terminus, nullusque supererit ubilibet hiatus; idque sine prævià quâlibet Reductione, depressione, limitatione, *&c.* omninò prout *Frontispicium* fusiùs indicat.

*Adrian Rom. Probl. Resp.* pag. 315.

Jam verò, si *Vieta*, sub intuitu inventionis duorum Theorematum (quæ sunt fundamenta omnis doctrinæ Angulorum Sectionum, ad rem verò nostram obliquè tantum spectantia) eâ extaticâ lætitiâ afficeretur, ut exclamaverit, Tibi, ô Diva Melusinis, oves centum pro unâ *Pythagoreâ* immolavi; liceat mihi (præ inventi hujus gaudio) qui non ovibus, quidem, ovationibus tamen cum Vietâ contendere. Absit autem, ut ipse mihi quicquam arrogem; quippe, si in hâc re Fortuna magis mihi (eruditionem neutiquam obtendenti) quàm aliis verè Lynceis & doctioribus arriserit, & aspiraverit; imo, si quivis alius

in

*For defect of which Dioristic method, I find ſaith he *) a great gap in Art, which to fill up, a mean Art is not ſufficient; which yet afterwards his moſt acute wit (I confeſs) moſt accurately filled up: Which whether it may be of ſo great uſe in reference to the Arithmetical Reſolution of Equations, I ſhall not here determine; but whether it be of ſo great moment and ſo abſolutely neceſſary (as he poſſitively affirms) to the* Geometrical *Conſtruction of Equations, I ſhall leave to the judgment of the more Learned, when they ſhall have peruſed this Treatiſe: Wherein I doubt not, but they will find the* Geometrical *Conſtruction of all Equations, howſoever affected, not exceeding the fourth degree, without the Midwifery of a Dioriſtic to be exactly performed; nay, I may add, without the help of Meſolabe, Section of Angles, whether the ſecond or any other term be abſent or not, without leaving any hiatus any where, without any previous Reduction, depreſſion, limitation, &c. altogether as the Frontiſpiece at large declares.*

* Loco citato.

*And now, ſhall* Vieta, *upon the review and proſpect of having found two Theorems (which indeed are the Fundamentals of the whole Doctrine of Angular Sections, but obliquely only reſpects our buſineſs in hand) be traſported into ſuch an extaſy of joy, as to cry out * ô Diva, Meluſinis, tibi oves centum pro unâ* Pythagoreâ *immolavi? And ſhall the Author for the joy of this invention, vye with him in his joy, tho he cannot in Hecatombs? But far be it from me, to arrogate any thing to my ſelf; for if in this (or any other) inſtance, fortune ſhould ſmile on me (who pretend not to Learning) more than on others of greateſt eminency in Learning; nay, if even they too ſhould*

* Adriani Rom. Prob. Reſp. pag. 315.

in inventum quodvis inclytum (nomen ſuum immortalitati mandaturum) fortuitò inciderit; non eſt quòd exinde vel gloriolam ſibi arroget, aut magis inſoleſcat, quàm qui fortè fortunâ uno & unico globi miſſilis projectu totum ſimul Conorum Luſoriorum Enneada dejecerit; aut quàm qui *Hercule* colludens caſu haud prorſus abſimili illum proſtraret.

Si verò mihi vel tantillum (cui ne hilum quidem) gloriæ, hujus inventi cauſâ, tribuendum; plurimum equidem celeberrimo Carteſio primò impetiendum lubens agnoſco; cujus humeris (velut nanus quidam) inſidenti, longius diſſita illò paulo acutiùs perſpexiſſe conitgit; à cujus face lucernula hæc (qualis qualis ſit) lucem ſuam (velut à Sole Luna) mutuata eſt.

Non nihil etiam (quod gratitudinis ergô, amicitiæque devinctiſſimæ ſpecimen refero) D. *Thomæ Strode* de *Maperton* in agro *Somerſetenſi*, viro verè generoſo, præſtantiſſimoque Mathematico meritò reddendum; non tantum eò quod reperti hujus anſam præbuit; ſed quod ejuſdem Ideas quaſdam ſubminiſtravit Lectiſſimus iſte (qui penes eum eſt, & quem mecum humaniter communicavit) liber *M. S.* In quo proprietatem ad Diametrum Parabolæ ſpectantem ſuprà memoratam reperi; quâ ſine malè forſitan ſucceſſiſſent omnia, parumque abfuiſſet, quin invento excideram: Libellus ſanè utiliſſimus, magnique æſtimandus, utpotè, qui non ſolum propriis novis & abditis, ſed præclaris omnibus ex intimis Authorum ferè omnium Sectiones Conicas tractantium viſceribus erutis, refertiſſimus; cujus ſubtiliſſimæ de Hyperbolicis ceteriſque Coniſectionibus Propoſitiones ſi pro merito prædicarentur cunctorum Rhetorum Hyperbolas ſuperarent.

Ad

*ſhould chance to light on ſome famous invention, which might in a more eminent manner immortalize their names; yet have they no more ſhadow of reaſon to be proud of it, than he, who accidentally at one tip, ſhould ſtrike down the whole pack of Nine-pins; or, than he, who in ſporting or dallying with* Hercules *ſhould by a like chance foyl him.*

*But if any praiſe were (which is none) due to me, for the invention; a very great ſhare muſt redound to the honour, firſt of the famous* Des Cartes, *on whoſe Gigantic ſhoulders ſtanding, I chanced to ſee further than he; but I confeſs, this Candle (ſuch as it is) was lighted at his taper.*

*Another part (which I mention as a ſpecimen (ſuch as it is) of my gratitude and reſpects) is deſervedly due to that moſt worthy Gentleman and moſt Excellent Mathematician Mr.* Thomas Strode *of* Maperton *in the County of* Somerſet; *not only for the occaſion given of this invention (beſt known to him only) but for the light I received from his incomparable* M. S. *touching Conical Sections; wherein I found the propriety belonging to the Diameter of a Parabole above mentioned; without which the invention it ſelf might perhaps have proved abortive. A Treatiſe of ſuch worth and uſe, that beſides his diſcovery of many new and hidden things never extant, it ſeems to have engroſſed all that is excellent in every Author (that I know) that hath treated or is extant on that ſubject; and which needs the Rhetoricians Hyperboles, to recommend to the World the excellency of his* Geometrical *ones, and other Conical Sections.*

*But*

Ad rem verò: Disquisitionis hujus exitus non magis fœlix faustusque fuit, quàm media (quibus hoc inveniendo usus sum) apprimè congrua; quæ ad calcem hujus Tractatûs apposui; eoque potissimùm consilio, ut non tantum te (Tyro) manuducerem, tibique animos adderem; sed ut iisdem (aut non absimilibus) premens vestigiis, propriis studiis & laboribus altiora & non priùs audita moliaris.

Si enim applicetur methodus nostra (quâ duce hæc invenimus) tam ad Hyperbolas & Ellipses, quàm ad Parabolas; imò ad paraboloeides, Hyperboloeides, Elliptoeides, sive ad quasvis alias elatioris gradus Curvas, quà similis, quà dissimilis Constitutionis (quas comminisci poterint studiosi) efformandas; haud dubiè particulares saltem (imò & universales) Regulæ, ad quamplurimas (etiam ad omnes) Æquationes, ad quodvis ulterius par graduum construendas, emergent. Quod studiosis relinquo; virisque ingenio perspicaci adnotasse sufficiat, operæque forsan erit pretium.

Cuipiam idcirco mussitanti vel suggerenti hoc ipsum Inventi hujus gloriam imminuisse, quòd casu repertum, regero. Quo ad finem, non abnuo; attamen quoad media (quibus usus sum) dico; excogitatâ ratione me ea composuisse, & ad finem propositum assequendum adeò apta nata, ut aptiora nequaquam possint excogitari; tantique (hujusce farinæ rebus indagandis) momenti, ut cuipiam κατὰ πόδας eadem insequenti, res nominis sublimioris hisce à nobis inventis adeò facillimè assequi contingat, ut locus obtrectationi non relinquatur.

Porrò,

*But to return: The event or end however was not more lucky, than the means used, suitable; which I have on set purpose discovered at the heel of this Treatise, as well (Tyro) for thy Encouragement in this Study, as for the further improvement (if I mistake not) of this Invention, by the industry of the more Learned, insisting in the same (or the like) method; by which means higher things may be discovered.*

*For assuredly, the application of this Method (which I have used in this discovery) to Hyperboles, Ellipses, as well (as to Paraboles; nay, to Paraboloeids, Hyperboloeids, Elliptoeids, or to any other Curves of an higher degree, either of a like or different Constitution (which the Studious may find out) will (undoubtedly) discover particular (nay I may add, universal) Rules, for the Construction of divers (nay, all) Æquations of the fifth and sixth; or of any other pair of higher degrees, which I shall leave to those that are Studious; to whom to have animadverted this, may suffice, and perhaps worth the while.*

*To any one therefore whispering or suggesting, that it is a diminution of the Glory of this Invention, that it was found by chance, I reply; As to the end or event, indeed, I deny it not; but as to the Medium's I used, I say, that they are so well suited for the attaining of such an end proposed, as none could be more suitable; and of so high a moment in the search of things of this nature, that if it be exactly pursued, things of an higher nature may so easily be attained, that it may retrieve the dishonour, and procure more glory, than the chance of the invention can diminish.*

Porrò, non ignoro, quòd non deerunt Catones Censorii, qui Tractatum hunc nimiæ prolixitatis insimulaverint, immo vellicaverint, suggerintque forsan, quòd aptiùs & peritiùs; si strictiùs & succinctiùs egerim; nempè, si duas vel tres solas Æquationes, totidem Demonstrationibus muniveram, & Schemata vel Figuras ad quatuor vel sex, totumque opusculum ad 4 (plus minus) Schedas perstrinxeram; quod satis abundè totum clarè & perspicuè reddidisset, immo & Lectoribus sagacioribus (emunctæ licet naris,) magis arrisisset. Non equidem inficias eo, palamquè profiteor, me in privatum usum totum intra unius Schedulæ cancellos coercuisse.

At hoc tibi velim (Lector) innotescat; hæc nequaquam Veteranorum, qui ex pede *Herculem*, vel ex ungue Leonem probè nôrunt, exarata; at Tyronum & sciolorum gratiâ, qui neque hunc vel illum, ex hoc, vel ex illo, nisi digito monstrati, dignoscere sapiunt. Eo igitur potissimùm instituto ista composuimus, ut illis commodo, usui & adjumento essent, qui aut nolunt, aut (cùm crassæ sint Minervæ) nequeunt concisam & Laconicam brevitatem capere.

Horologium tam mirè fabrefactum (quale quondam Reginæ dono datum) ut præangusto annuli signatorii ambitu latitaret, utut Artificis solertiam & ingenium mirificè deprædicet, suæ tamen magis famæ, quam Reginæ usui consuluisse, nullus dubito.

Annon

*Moreover, I am not ignorant, that ſome cenſorious* Cato*'s may implead and impeach this Treatiſe of too much prolixity; and will perhaps ſuggeſt, I had done more like an Artiſt, if I had been more ſuccinct and conciſe;* viz. *If I had fortified two or three (at the the moſt) Æquations with as many Demonſtrations; and had confined the Schemes or Figures to* 4 *(more or leſs) and the whole, to as many ſheets, which would have rendred the whole ſufficiently intelligible, yea, and have better gratified the moſt curious Reader. True: I deny it not, and ingeniouſly confeſs, that for my private uſe, I have confined and contracted the whole to one ſheet.*

*But know this (Reader) withal; this was never deſigned by me, for ſuch as are Veteranes, perpolite Artiſts, who know Lions by their paws only, and* Hercules *by his foot; but for young, ſlow Mathematic Scioli's who know neither a Lyon nor* Hercules, *unleſs they are told ſo. I deſignedly ſpun it into a long thred, for the eaſe, uſe and encouragement of ſuch, who either will not take the pains, or have not the brains to apprehend ſuch Laconic conciſeneſs and brevity.*

*A Watch contrived within the narrow ſphere of the ſignet of a Ring (which was once preſented unto a Queen) may commend the skill and ingenuity of the Artificer; yet its uſefulneſs never as yet recommended it to the Worlds uſage.*

Annon *Homeri* Ilias literis majusculis, folioque edita lectu (ideoque intellectu) faciliora; Annon *Johannis Tredescanti* cochlearia argentea vulgaria justæ magnitudinis utiliora; quam cùm hæc nuci, illa Cerasi ossiculo impacta aut damnata?

Annon *Euri* leniter spirantis mollis aura genus humanum magis recreat & refocillat; quàm quæ Globo *Ithaco*, vel Philosophico conclusa?

*Cartesio* ipsi, quamvis etiamnum plurimos illum meritò demirantes detinet; plures tamen Lectores, paucioresque Commentatores procul dubiò habuisse contigerit; si non ex professo * (ut fatetur) concisus succinctusque fuisset.

* *Geom.* l. 3. p. 105.

Sæpissimè animadverti, Brachygraphica concionum Adversaria subitò repetendarum utilissima fuisse; attamen temporis progressu, Authores ipsos non esse relegendo, nedum intelligendo; vilissimisque (quibus solis tandem idonea comperta sunt,) (horresco referens) usubus addicta.

Alia insuper gravis causa me inpulit paulò fusiùs dilatandi. Observanti enim mihi occurrit, opuscula quævis plus satis concisa (elegantissima licet & concinna) eo ipso nomine rariùs vænire; quia sc. paucissimorum captui (quod maximè nunc dierum præcavendum) sunt attemperata. Non enim de libris Mathematicis, quod de *Româ* quondam asserendum; sc. Omnia *Romæ* esse vænalia.

Hujus

*Are not* Homer's *Iliads Written in Capital Letters and enlarged unto a Folio, better legible (and therefore the more intelligible) and* John Tredescant's *common Silver House-spoons more useful, than when the one are crammed into a Nut-shel; and the other, into a Cherry-stone?*

*Do not* Eurus's *gentle soft blasts refresh and cool more, then when imprison'd within the Concave of the* Ithacan *(or Philosophers) ball?*

Des Cartes *himself, had he not been so designedly concise and curt (as himself says he was *) tho he hath still many Admirers, yet might he have had more Readers, and fewer Commentators.*

* Geom. l.3.p.105.

*And I have often observ'd, tho Brachygraphical Sermon-notes have proved very useful for a sudden Repetition; yet after some years have been as illegible, and unintelligible to the very Pen-men of them themselves, as useful for some other employ, (which I tremble to relate) to which themselves have condemned them.*

*Besides, another weighty reason induced me, to enlarge on this Subject; for I have observed, too much conciseness in any Treatises of this kind (tho embellisht with never so much Elegancy, Art, and Concinnity) even in that respect, renders them the less vendible;* viz. *because, not suited (which had need now-a-days, be aforehand considered of) to the capacity of the vulgar: What of old was attributed to* Rome, *may not now be attributed to Mathematic Books,* viz. *Omnia Romæ sunt venalia.*

*For*

Hujus equidem ævi moris non est, inopes, & ætatis provectioris (etiamsi pulchras) viduas, (qualis est *Mathesis*) (quæ tamen eò pulchrior, quò natu grandior) absque summâ dote sibi desponsare; quâ quidem (& non contemnendâ) Tractatulum hunc (perspicuitate licet, quoad potui, adornatum, quò magis vænalis redderetur) cumulari necesse priùs erat, quàm sponsorem adipisci. Τὸ Quantum dabitis, (si cum Typographis res habenda sit Mathematica) primâ facie auribus injicitur, objiciunturque protinùs oculis quamplurimi exoleti, exesi, & tineosi hujus farinæ libri, invenundati, & (uti inquiunt) invendibiles: Si indotata Mathesis, facesse, hinc te ocùis proripe.

Aut si typis quævis id genus mandentur, si minus perspicua, Luculenta, & intellectu facillia, mercis resectanæ ad instar, in Bibliopolarum officinarum abacis obscuris, vel angulis sentis situ reponuntur. Quâ de causâ, (commonefactus etiam ab illis, quibus morem non gerere grande esset piaculum (opusculum istud prodire bilingue necessum erat. Quibus rationibus adductus (Tyro Philomathematice) tuo etiam commodo omninò consulens, tuoque modulo omnia attemperans, Deam hanc cultu prorsus vulgari (syrmate licet paulò productiore) vestitam amplectere; cujus reverà non est (Rhetorum instar) bombycinis *Tyriis* adornari: Non enim convenit eis, qui in perpetuâ veritate versantur, ampullas projicere, & sesquipedalia verba.

Age

*For it is not the guise and humour of this Age to espouse poor, Aged (tho fair) widowed Ladies (as (*Mathesis *is) (who yet, is by so much the fairer, by how much the more ancient) without a considerable Dowry: Which even this Treatise (tho it hath, the accession (as much as I could) of perspicuity and plainness to advantage its sale) hath had (and all others, of the like Complexion) must have) ere an undertaker would, or can be had. To whom if you apply your self, and the concern be Mathematic, you must expect your Ears to be stormed with a* Qantum dabitis: *And presently produced to your view an infinite multitude of Exolete, half-moth-eaten Books, of this kind, unsold, and (as they will perswade you) never will.*

*If no portion farewel; or if one, and it be Printed, it must be clad in such a plain Garb or Dress, as may render it easie to be understood; or else, like braided wares in Shops, they will be placed in some nasty Corners, and lye upon the Book-sellers hands. For which reason I was enforced also, and advised too by some (whose desires are commands) to make it double-tongued. Thus (kind Reader) upon these inducements, consulting they ease and profit, and having suited all things to thy capacity, embrace this Goddess, tho clad in a long robe, yet in a plain dress. Her nature inclines not to be arrayed (like Rhetoricians) in* Tyrian *Silks: For it is very unsuitable to those, who are conversant in perpetual Truths, to project for bombast Language.*

*Sit*

Age ergò (Lector) ad mensam Lucubratoriam sedeas; in Tabulâ Centrali sive Synopsi Æquationem congruam, cujus Constructionem velis, quæras, quæ ad Regulam, Demonstrationem, Figuram illi accommodam diriget: Circinum sume scalamque digitorum (ea enim est, quâ κατὰ πάντα usi sumus) descriptâque secundum artem (prout capite hujus Tractatûs te edocuimus) Parabolâ, cujus latus Rectum sit unitas, vel digitus unus, instituantur & applicentur omnia, juxta ibidem præscripta; omniaque votis tuis ad amussim responsura, è vestigio reperies.

Si serenâ fronte & ambabus ulnis hæc (qualia qualia) te excepisse noverim, ad altiora meditanda stimulos adjicies; quæ quidem jamdudum calamus ad umbilicum perduxit; sin minùs æternùm de Tabulâ.

Spero (Lector) te Errata, quà Authoris, quà Typographi (si quæ irrepserint) neutiquam ægrè Laturum: (quæ tamen (si quæ sint) vel in Tractatu vel in Figuris) facillimè inter se conferendo, possint corrigi) cùm rem suam longo nimis intervallo ab invicem dissiti, uterque egerit.

*Vale.*

*Sit down therefore at thy Study-Table (Reader) ſeek the Æquation whoſe Conſtruction thou deſigneſt, in the Central Table, or Synopſis, which will guide thee, to its Rule for its Conſtruction, its Demonſtration, Figure, or (at leaſt) to one ſuitable to it. Take thy Compaſs and the Scale of Inches (for that Scale only have I uſed through the whole) and having deſcribed according to Art (which in Chap. 1. is taught) a Parabole, let all things be applied accordingly, as we have preſcribed; and thou ſhalt find all things forthwith exactly to anſwer thy expectation.*

*If this (ſuch as it is) be kindly accepted at thy hands; as it will encourage me to meditate on things of a ſublimer nature, ſo will it to publiſh thoſe, which I have already finiſhed; if not, this is too much, which I have already done.*

*I hope (Reader) thou wilt be ſo Candid and Juſt, kindly to pardon; as the Lapſes (if any) of the Author, ſo of the Typographiſt, (which yet, both in the Treatiſe and the Schemes, may eaſily be Corrected, by comparing one with the other) who did his buſineſs at too remote a diſtance from the Author.*

Farewell.

## *Notæ seu Symbola, quibus in sequentibus utor.*

Additionis $+$

Subductionis $-$

Cum non proponitur utra Magnitudo sit Major, vel Minor, & tamen subductio facienda est nota Differentiæ est $\backsim$, id est, Minus incertò; ut proposita AO $\backsim$ AD; Differentia erit AO — AD; vel AD — AO.

Multiplicationis, $\times$

Æquale, $=$

Majus $\sqsupset$

Minus $\sqsubset$

Parallela, $\parallel$

Perpendicularis, $\perp$

Quadratum, Q.

Quadratum Radii, Q. Rad.

Quadratum x, $x^2$

Quadratum NO, $NO^2$; &c.

Quadraticè involutum, $\ominus$

Cubus x, $x^3$

Quadrato-quadratum x, $x^4$

Ratio, sive proportio : :

Vel, u, Ergo, g°.

## *The Explication of the Notes or Symbols.*

Addition $+$

Subduction $-$

The Difference between two quantities, when it is not propounded which of them is the Greater or Lesser, and neverthelefs the Subduction is to be made, $\backsim$

Multiplication $\times$

Equality $=$

The Greater $\sqsupset$

The Lesser $\sqsubset$

Parallels SS

A Perpendicular $\perp$

The Square, Q.

The Square of the Radius, Q. Rad.

The Square of x, $x^2$

The Square of N O, $NO^2$, *&c.*

The Involution of the Square $\ominus$

The Cube of x, $x^3$

The Quadrato-quadrat of x, $x^4$

Ratio, or Proportion $::$

# *A Catalogue of the Mathematical Works of the Learned Mr.* Thomas Baker, *Rector of* Bishop Nympton *in* Devonshire, *with a Proposal about printing the same; and first one, intituled, The* GEOMETRICAL KEY, *or the Gate of Æquations Unlockt.*

A New Discovery of the Constructions of all Æquations howsoever affected, not exceeding the *4th* degree; *viz.* Of *Linears, Quadratics, Cubics*, and *Biquadratics*, and the finding of all their Roots as well true as negative, without the use of *Mesolabe*, and *Trisection of Angles*; without Reduction, Depression, or any other prævious preparation of Æquations, by a Circle and any (and that but one only) *Parabole*: and this by one only general Rule, than which a more simple, more perfect, more general, more easy to be understood, or more fit for practise, cannot be devised or wished for.

Fortified with Demonstrations, Illustrated with Figures to each Æquation, which are Exemplified with numeral Æquations, (according to all the varieties of Cases) adapted to each Figure, for the use of young Mathematicians: a work hitherto desired.

THe Treatise consists of about a Quire of Paper, the Discourse whereof (but not the *Algebraick Calculus*) is both in *Latin* and *English*, the better to promote its forreign vend; and this doth not render it above three Sheets the larger than it would have been in one of these Languages. Besides which, there is belonging to it diverse Draughts of Schemes to be engraven, and one *Folio* Draught, whereto the literal *Calculus* for setling the *Center*, and finding the

*Radius* of the Circle that is to intersect the *Parabole* is expressed in readiness for all Cases.

How *Des Cartes* and all other famous *Analysts* came to miss this general Rule, and himself to fall upon it, he acquaints the Reader in the middle of his Discourse; namely, that they considered the Axe of a *Parabole* and not its Diameter: and affirms, that if it had been his or their hap to have described a Circle from any Point in *Position* given, passing through the *Vertex* of any Diameter in the *Parabole*, and had taken into consideration a certain propriety (than which none could so have suited the design) belonging to the Diameter of any *Parabole*, they could not but with greatest ease, have made a full discovery of the Universal Rule.

The excellency of which Invention appears, in that it discovers not only the Geometrical Construction of all Æquations as abovesaid, by one only standing measure and Scheme, and that by one only general rule, with the exact number of Roots as well true as negative, but also by giving a fair prospect towards their *Arithmetical Calculus*, or numerous Resolution, by making a Discovery of their two first figures or numbers; namely, by applying the Compasses to the several Roots Geometrically found in the Scheme, and comparing them with that very Scale from which the said Scheme (suited to the proposed Æquation) was drawn, the residue of which roots, (though not precisely, yet sufficient nearly approximating to the true) may diverse ways in Decimals be found out, which the Author (as he intimated in a Letter of *April* 1682, to Mr. *Collins*) is willing to impart; but as to the Invention of these residuals (to be entail'd to the two first figures or Numbers of this Author thus findable.) The Learned Mr. *Isaac Newton* Professor of Mathematicks in *Cambridge* (in a Letter long since communicated to the aforesaid Mr. *Collins*) hath as to this purpose performed the same (as is conceived) by a different method, namely, that when a root of any Æquation is by any Method (which by the Authors aforesaid it may be) so near found, that it doth not differ above a tenth part of its self from the true root sought, the residue of the root inquired will be easily calculated by aid of some terms or Fractional parts of an infinite Series or rank of continual Proportionals, derived from the difference between the Resolvend of the known part of the Root, and that whose Root is sought. By which means by raising Resolvends out of any assumed Roots with an easy approach, without raising

the respective powers of the said Roots, we are delivered from the most toilsom Drudgery of Mathematical Calculations, by finding the Roots of Æquations in numbers, by *Vieta's* general method; a thing utterly unknown to the Ancients. However this is not said to disparage that Method which *Vieta* so greatly esteemed, that when he had obtained it, he gave *Algebra* this high *Encomium*, that it did *Nullum non Problema solvere*, in his numerical Method Mr. *Oughtred* and *Harriot* have taken commendable pains. But now last of all, to perform it in Species as Mr. *Isaac Newton* hath done, seems a new Invention never to be sufficiently praised; for out of a literal Æquation of five Dimensions, supposing all the terms extant and affirmed, he hath given a Series for the Root in Species, and such a one as shall serve for finding the Roots of all Equations of 3, 4 or 5 Dimensions, by only altering the signs according as as the Æquation is affected, and expunging such parts as relate to Deficient terms in an incompleat Æquation proposed.

Now that this admirable Doctrine may come to light, and the Learned Author (who hath many other Treatises worthy publick view) may be incited to impart the same, encouragements for the promoting thereof (seeing Undertakers are not to be had without) must be propounded.

It is therefore humbly offered, that the Royal Society by their Treasurer *&c.* enter into Bond to such Bookseller as shall be the Undertaker, to take off 60 of these Books in Quires at $1\frac{1}{2}$ *d.* each Sheet, and as much each Plate, as soon as printed.

*The Treatise it self and the Proposal, is approved and agreed to by the Council of the Royal Society.*

And in regard such a Subscription is not sufficient to incite an Undertaker, that the respective Members endeavour by virtue of this Narrative, to obtain as many more Subscribers as they can procure amongst others that are not of the Society, each of them to advance half a Crown in hand, in part of the former price: upon which encouragements, *Robert Clavel* Bookseller at the *Peacock* in St. *Pauls-Church-Yard*, is ready to give reciprocal security for the performance according to this Proposal, hoping the like encouragement will be given towards printing the rest of the Treatise of this most Learned Author, whereof take the ensuing Catalogue.

1. The

1. The *Hyperbolic* Key, or the Geometrical Construction of *Cubic* and *Biquadratic* Æquations; by a Circle and an equilateral *Hyperbole*; to wit the one moiety (exactly) *viz*. of eight *Cubicks* and four and twenty *Biquadratics*; (as is expressed in the former treatise) namely,

1. Of all those Cubicks; wherein

1. The quantity ($q$) is wanting; and $p$ and $r$ affected with divers Signes.

2. The quantity ($p$) is wanting, and in the Æquation be had $-q$.

3. None of the terms are wanting, and in the Æquation be had $+q$,

2. Of all those *Biquadratic* Æquations in which are had $+s$. by the demission of Perpendiculars, from the points of Intersections of the Circle and Hyperbole to the Assymptote; part of the other moiety, by the demission of perpendiculars from the aforesaid intersections to the axe; *&c.* with Schemes adapted to each Æquation, *&c.* with a Synopsis of the whole, wherein the Literal Rule for fixing the Center, and finding the Radius of the Circle, that is to intersect the Æquilateral Hyperbole (the easiest way of the Construction of which is likewise therein discovered) is expressed in readiness for all Cases.

This method of Construction (were it not, that for every Case a new Hyperbole must be described) would not be inferior to that by a Parabole, but rather exceed it; in that the Circle doth not arcuate the same way which the figure doth, but crosses it the other way; by which means a clearer discovery (as to the one moiety) of the points of intersection of the Circle with the Hyperbole is obtained, than what can possibly be had in any other Coni-section.

3. The Geometrical Construction of some Æquations which ascend to the *5th* and *6th* power, with the finding of their Roots, by a Curve of the third degree; namely by the first kind (for there are two kinds) of a Paraboleid and a Circle, illustrated with Schemes to each Æquation, and numeral Æquations adapted to them; with a Synopsis to the same for placing the Center and finding the Radius, and a general little Table, for the describing of both kinds of Paraboleids.

3. The

4. The Construction of all Cubick Æquations howsoever affected by a Circle only, Geometrically upon Supposition, that one *Postulatum* be granted to be Geometrical (which indeed is but a Supplement to Geometrical defects;) *viz.* That from any point assigned in the circumference of a Circle (that is normaly quadrisected) may be drawn a right Line, so that the parts intercepted both ways by the Circumference and Diameter, may be equal to the Radius of the Circle: this way (though not so purely Geometrical as the rest) is not to be despised, sith that these Lines may sufficient precisely be so drawn.

5. The Geometrical Construction of all Cubic Æquations according to the Rule found out by *Franciscus a Shooten*, mentioned in his Commentaries on *Des Cartes*, *Lib.* 3. *Pag.* 328, 329, 330, illustrated with Figures and Numeral Æquations adapted to each Figure, *&c.*

6. The Resolution of all Cubick Æquations in numbers, not only by a general Rule by the assistance of any Figure resolving them Geometrically *&c.* but by a more particular method far exceeding any extant in Numbers or by help of Tables; illustrated with Figures and Examples in numbers, suited to each figure and Æquation.

7. Mixt or Compound *Trigonometry*; in many instances far exceeding the simple, as finding two *Quæsitas* (as it were) by one operation, or by two at most; with a Synopsis of the admirable harmony between Plain and Sphærical Triangles: for instance,

In plain Rectangular Triangles, the ▭e under half the sum of the *Hypotenuse* and one side: and half their difference, is equal to the Square of $\frac{1}{2}$ the other side, so in Sphærical Rectangular Triangles. The ▭ under the Tangents of half the sum and half the difference of the *Hypotenuse* and one side, is equal to the square of the Tangent of half the other side.

Again, in Obliquangular Plain Triangles.

$$\frac{cs,}{s,}\ \tfrac{1}{2} Z \angle\angle : \frac{cs,}{s,}\ \tfrac{1}{2} X \angle\angle : \tfrac{1}{2}\ \mathit{Basis} \begin{cases} s\ \tfrac{1}{2},\ Z\ \mathit{crurum.} \\ t\ \tfrac{1}{2},\ X\ \mathit{crurum.} \end{cases}$$

Thus likewise in Sphærical Obliquangular Triangles.

$$\frac{cs,}{s,}\ \tfrac{1}{2} Z \angle\angle\ \mathit{ad\ Basem} : \frac{cs,}{s,}\ \tfrac{1}{2} X \angle\angle\ \mathit{ad\ Basem} :: \tfrac{1}{2}\ \mathit{Basis} : \begin{matrix} t,\ \tfrac{1}{2} Z\ \mathit{Crurum.} \\ t,\ \tfrac{1}{2} X\ \mathit{Crurum.} \end{matrix}$$

Again in plain Triangles.

$\frac{1}{2}$ *Basis.* $\frac{1}{2}$ Z *crurum* : $\frac{1}{2}$ X *crurum.* $\frac{1}{2}$ X of the Segments of the Base.

In

In Sphærical also.

$t, \frac{1}{2}$ *Basis*. $t, \frac{1}{2}$ Z *Crurum*, $t, \frac{1}{2}$ X *Crurum*, $t, \frac{1}{2}$ X *Segmentorum basis*, with infinite other alike harmonious.

To which is added the Geometrical Construction of all Sphærical Triangles, by a most plain and easy uniform way, which is indeed of singular use.

Also a discovery of the Method by which *Vieta* (*Lib*. 8 *p*. 431 *&c.*) found out his Canonical Analogy of Sphærical Triangles, which he hath left undemonstrated, but in this Treatise is discovered.

8. *Cardanus Promotus*, or *Cardans* Rules, or *Vieta's duplicata Hypostasis*, *in infinitum*, carried on with a Table for the composition *in infinitum* of such Æquations. By which means such Canons are generally composed for Æquations of two Nomes (and in many Cases for more) equal to a Resolvend given.

9. A Continuation of *Vietas Apollonius Gallus*, *Appendicula*; 1. And his Problemes otherwise demonstrated, wherein the Base and Angle opposite to the Base are always two of the *Data's*, and the other, either the perpendicular or the difference of the Segments of the Base, or the difference of the squares of the sides, or the sum of the Squares of the sides, or the Sum of the sides, or the difference of the sides, or their ▭, whose Geometrical effection was altogether unknown to the antient Analysts, *Vieta ibid.*

10 *Vera & Genuina Symmetrica Climactismus*, by which means all *Asymmetries* in *Algebraics* may be wiped off, and an Æquation found in any one of the unknown magnitudes proposed, which shall never ascend higher, than the double of the highest power first proposed, by which also that most perplexing entangling inextricable way of *Vieta* may be laid aside as useless, and inefficacious, though hitherto it hath been the only remedy. *Adversus vitium Asymmetriæ*, this treatise was many years since composed and laid aside; but the Author lately meeting with the *opera Posthuma* of *Monsieur de Fermat*, treating on the same Subject in his (*Varia opera Mathematica pag*. 58, 59, &c.) and finding that though he rightly hits the mark; yet that he goes not in a streight Line to it, hath revised his old Copy, and compared it with *Fermats*; and which of the two, hath gone the Simpler way, the Author leaves to the judgment of others, being loath in the least to take up the Gantlets against such a famous man whom the world admires.

11. *Apollonius*

11. *Apollonius magnus Gregorianus*, or a Treaiise of four Geometrical proportionals, wherein divers ways are found to solve that Grand problem, which hath so amused the world, (*viz.*)

Having the sum of all the Squares, and the sum of all the Cubes, of four Geometrical Proportionals to find the Proportionals themselves; with questions of the like nature, by low Æquations, without aid of Analytick Store.

12. Of Triangular Sections by a different method than what *Anderson* has performed it by, in *Vieta*, with a discovery of the falshood (as to angular Sections) of Mr. *Oughtreds* *1st.* Rule, in his *Clavis Mathem. c.* 16. *p.* 14.

13. The finding out of Æquations which may infinitely ascend, whose Roots are either in Arithmetical or Geometrical proportion which may be found out in numbers by extracting the Square and Cube Root, with surd Canons adapted to that purpose, and to many other Æquations.

14. A Miscellany of the solution of many knotty Problemes, namely, such as have been found difficult to be brought to any Equation, or else would mount very high *in Ordine Scala*, with a new method of Depressing them, by aid of one or two Æquations, raised by altering the Data, and putting two unknown quantities, by which means the adjutant Æquations as having the same common root, depress the Æquation that otherwise should be resolved

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## *ADVERTISEMENT.*

THe Author herein supposeth the Reader to understand the use of common Symbols described in his first Book; *viz.* cs, for *Cosine*, s, for *Sine*, Z for *Sum*, X for difference, ∠ for *Angle*, ∠∠ for *Angles*. And the Reader must be informed, that as the whole seems novel, so a brief Demonstration of those Proportions in *Sec.*7. to hold in Sphæricals is most desirable; and if others be not wanting in their encouragements, it's not to be feared the Royal Society will be slow in theirs.

11. *Apollonius magnus Gregorianus*, or a Treatise of four Geometrical proportionals, wherein divers ways are found to solve that Grand problem, which hath so amused the world, (viz.)
Having the sum of all the Squares, and the sum of all the Cubes, of four Geometrical Proportionals to find the [illegible] [illegible] questions of the like nature, [illegible] quations, without [illegible] Analytick Stone.

12. Of Triangular Sections by a different method than what *Anderson* has performed it by, in *Vieta*, with a discovery of the falshood (as to angular Sections) of Mr. *Oughtred's* [illegible] Rule in his *Clavis Mathematicae* 6, c. 14.

13. The finding out of [illegible] Equations whose whole Roots are either in [illegible] tion which may be found out in numbers by extracting the Square and Cube Root, with said Canons adapted to that purpose, and to many other *Aequations*.

14. A Miscellany of the solution of many knotty Problems, [illegible] natural [illegible] found difficult to be brought to any Equation, [illegible] would [illegible] very high a [illegible], with a new method of Depressing them, by aid of one or two Equations, raised by altering the Data, and putting two unknown quantities, by which means the adjunct Equations having the same common root, [illegible] otherwise [illegible]

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The Author herein supposeth the Reader to [illegible] of common symbols [illegible] described in his [illegible] Co-sine, s. for Sine, Z for Sum, X for difference, [illegible] for Angle, ∠ for Angles. And the Reader must be informed, that as the whole seems novel, so a brief Demonstration of those Proportions [illegible] to hold in Sphericals is [illegible], and if [illegible] be [illegible] ing in their encouragement [illegible] ry will be shewn [illegible]

*Clavis Geometrica Catholica:*

# THE GEOMETRICAL KEY, OR THE Gate of Equations unlock'd.

# *Clavis Geometrica Catholica.*

Prænoscenda sunt quædam cognitu quidem admodùm necessaria; nempè

Natura ⎫
Proprietates ⎬ Parabolæ.
Constructio ⎭

## C A P. I.

*Methodus Synthetica postulat, ut à Parabolâ, tanquam universali hujus Tractatûs Subjecto, ad partes, hoc est, ad Rectas quæ in Parabolæ partibus expendantur, tanquàm principia & causæ; tum demùm ad Parabolæ affectiones sive proprietates; (ut tandem ad ejus Constructionem) fiat processus.*

*Fig.* 1. CUilibet, vel in limine Mathesin salutanti notum est, Basem *(bNc)* Coni *(abc)* esse circularem, punctumque *(a)* vocari ejus Verticem; & Rectam *(aZ)*, à Vertice *(a)* ad centrum Baseos Circularis *(Z)* perductam), appellari ejus Axem. Quibus agnitis,

*Fig.* 1. 1. Si Conus *(bac)* plano secetur per Axem *(aZ,)* resultabit Triangulum *(abc;)* in cujus Plano ducatur recta *AO* (cuilibet laterum, puta) lateri *(ac)* parallela. In plano Baseos Circularis *(bNc,)* erigatur ad Diame-

# THE GEOMETRICAL KEY.

## Some Things truly very necessary to be known, are to be premised; *viz.*

The { Nature, Properties, Construction } of a Parabole.

## CHAP. I.

*Synthetical Method requires, that we proceed from a Parabole, as the universal Subject of this Treatise, unto the Parts, that is, unto those Right Lines, which are considered in the parts of a Parabole, as the principles and causes; then at length, unto the affections and properties of a Parabole; that so way may be made for its construction.*

IT's well known to every mean Mathematician, that the Base (*b N c*) of the Cone (*a b c*) is circular; and the Point (*a*) is called its Vertex, and the Right Line (*a Z*) (which is drawn from the Vertex (*a*), to the center of the Circular Base (*Z*,) is termed its Axe. Which being known, Fig. 1.

1. If the Cone (*b a c*) be cut with a Plane through its Axe (*a Z*,) there will result the Triangle (*a b c*;) in whose Plane, draw *A O* parallel (to either of the sides, suppose) to the Side (*a c*.) In the plane of the Circular Fig. 1.

Diametrum $(bc,)$ perpendicularis $ON$. Sectus autem sit idem Conus altero plano secundo, secante Basim Circularem $(bNc)$ secundum duas Rectas $ON$, $OA$. Sectio curva $(AN\Theta R)$ resultans vocatur *Parabola*.

1. 2. Recta $AO$, (quæ quidem omnes Lineas quæ in Parabolâ ducuntur sibi invicem Parallelas (ut $NR$, $nR$, $NR$,) bifariam dividit (in $O$, $o$, $o_1$) dicitur *Parabolæ Diameter*. *Fig*. 1.
2. Et si Recta $AO$ (omnes prædictas Parallelas bifariam dividens) ad Angulos Rectos secet, vocatur *Axis* (sive *Diameter originaria*:) Sin ad Obliquos, vulgò (absque ullo alio additamento) dicitur *Diameter*.
3. Unaquæque Rectarum sibi invicem Parallelarum, abs Axe vel Diametro bifariam divisarum (nempe, $NR$, $nR$, $NR$,) vocatur usitatiùs *Ordinata*; hoc est, Recta ad Axem vel Diametrum ordinatim applicata.
4. Portio verò Axis vel Diametri (ut $AO$) inter ordinatam ($noR$), & Verticem Parabolæ ($A$) intercepta, vocatur *Abscissa* Axis vel Diametri.

3. Parabolæ affectiones sive proprietates (quæ ad rem nostram spectant) ad hunc modum possint expiscari. *Fig*. 1.

Supponamus eundem Conum $(abc)$ sectum esse Plano tertio $(DnH,)$ Basi Circulari $(bNc)$ parallelo; liquebit, fore

| | | |
|---|---|---|
| | 1 | $oH = Oc$; (ob $ac \parallel AO$; & $bc \parallel DH$.) |
| *Ob Circ.* | 2 | Et $bO \times Oc = ON^2$; & $Do \times oH = no^2$. |
| $\triangle$ *Sim.* | 3 | Et $\left\{ AO \,.\, bO :: Ao \,.\, Do \right.$ |
| 1. | 4 | $\left. Oc = oH \right\}$ multipl. |
| $3 \times 4$ | 5 | $AO \,.\, bO \times Oc :: Ao \,.\, Do \times oH$ |
| 5, 2 | 6 | h. e. $AO \,.\, ON^2 :: Ao \,.\, no^2$. |
| *Inversè.* | 7 | $AO \,.\, Ao :: ON^2 \,.\, no^2$; quæ est propr. generalis. *Fig*. 1. |
| 7. | 8 | $\dfrac{NO^2}{AO} = \dfrac{no^2}{Ao}$; (quarum quævis sit) $=$ L, quæ dicitor Latus Rectum. |
| 8. | 9 | g°, $L \times AO = NO^2$; & $L \times Ao = no^2$. |
| 9. | 10 | h. e. $\left\{ \begin{array}{l} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao \end{array} \right\}$ Atque hæc est proprietas prima specialis. |

Base *(bNc,)* erect $ON$ perpendicular to the Diameter *(bc;)* and let the same Cone be cut with another second Plane, cutting the Circular Base *(bNc)* according to the two Right Lines $ON$, $OA$. The crooked Section *(ANOR)* resulting, is called a *Parabole*.

1\. 2. The Right Line $AO$, (that which bisects all parallel Lines in a Parabole, (as $NR$, $nR$, $NR$,) in the points $(O,o,o,)$ Fig. 1.
2\. is called, *The Diameter of the Parabole.* And if the Right Line $AO$ (bisecting all the aforenamed Parallels) cuts them at Right Angles, then is it called, *The Axe*, (or, *The originary Diameter:*) But if at Oblique, then (without any other Additament) it is usually called, *The Diameter*.

3\. Each of those abovenamed Parallels, bisected (as abovesaid) by the Axe or Diameter, is called usually, *An Ordinate*; *i. e.* a Right Line ordinately applied to the Axe or Diameter.

4\. But that portion of the Axe or Diameter (as $Ao$), intercepted between the Ordinate $(nok)$, and the Vertex of the Parabole $(A,)$ is called, *The Abscist*, of the Axe or Diameter.

3\. Those affections or properties of a Parabole, which concern our matter in hand, may thus be found out. Fig. 1.

Suppose we the same Cone $(abc)$ to be cut with a third Plane $(DnH,)$ parallel to the Circular Base $(bNc:)$ It will be evident,

| | | |
|---|---|---|
| | 1 | $oH = Oc$ (for $ac \parallel AO$; and $bc \parallel DH$.) |
| *Circle.* | 2 | And $bO \times Oc = NO^2$; and $Do \times oH = no^2$. |
| | 3, 4 | And $\left\{\begin{array}{l} AO \;.\; bO :: Ao \;.\; Do \\ \;.\; Oc = oH \end{array}\right\}$ multiply. |
| $3 \times 4$ | 5 | $AO \;.\; bO \times Oc :: Ao \;.\; Do \times oH$. |
| 5, 2 | 6 | h. e. $AO \;.\; NO^2 :: Ao \;.\; no^2$. |
| *Inverse.* | 7 | $AO \;.\; Ao :: NO^2 \;.\; no^2$; which is the general property. Fig. 1. |
| 7. | 8 | $\left\{\dfrac{NO^2}{AO} = \dfrac{no^2}{AO}\right.$; (let each of them be) $= L$, which let be called, *The Right Side*. |
| 8. | 9 | g°, $L \times AO = NO^2$; and $L \times Ao = no^2$. |
| 9. | 10 | i. e. $\left\{\begin{array}{l} L \;.\; NO :: NO \;.\; AO \\ L \;.\; no :: no \;.\; Ao \end{array}\right\}$ And this is its first special property. |

11

9 11 $\frac{Ny^2}{L} = ay$

9 12 $\frac{Ae^2}{L} = ae$ Fig. 2.

11 ∽ 12 13 $\frac{Ny^2 \backsim Ae^2}{L} = \left\{ \begin{array}{c} \frac{\overline{Ny+Ae} \times \overline{Ny-Ae}}{L} \\ \frac{NO \times OR}{L} \end{array} \right\} = \left\{ \begin{array}{c} ay \backsim ae \\ AO. \end{array} \right\}$

14 14 $\begin{array}{l} g^{\circ}, \\ u; \end{array} \left\{ \begin{array}{l} L \,.\, NO :: OR \,.\, AO \\ L \,.\, no :: oR \,.\, Ao. \end{array} \right\}$ Quæ eſt proprietas ſecunda ſpecialis.

Proprietas Parabolæ generalis (§ 7.) verbis enunciatur ſic; nempè, $AO \,.\, Ao :: NO^2 \,.\, no^2$.

Quadrata Rectarum (*NO*, *no*,) ad Axem (vel Diametrum) ordinatìm applicatarum, diſtantiis ſuis à vertice (*A*;) vel, (quod perinde eſt) Quadrata Ordinatarum (*NO*, *no*,) Abſciſſis ſuis (*AO*, *Ao*,) ſunt directè proportionalia.

1. Parabolæ proprietas prima ſpecialis verbis enunciatur ſic. (§ 10.)

ſc. $\left\{ \begin{array}{l} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao. \end{array} \right\}$ Fig. 1.

Recta (*NO*) ad Axem ordinatìm applicata, eſt media proportionalis, inter Abſciſſam ejus (*AO*) & Latus Rectum (*L*.)

Vel, Ut Abſciſſa (*AO*,) eſt ad ejus Ordinatam, cujus eſt Abſciſſa (*NO*;) ita ipſa Ordinata (*NO*,) eſt ad Latus Rectum (*L*.)

2. Secunda verò proprietas ſpecialis (§ 14.) ſic:

Si ad Axem Parabolæ (*ay*,) ordinatìm ſint applicatæ duæ Rectæ (*NR*, *BA*;) Dico,

Ut Latus Rectum (*L*,) eſt ad aggregatum ipſarum Rectarum; ita earundem differentia, ad differentiam ipſarum Abſciſſarum; nempe, $L \,.\, \left\{ \begin{array}{c} Ny+EA \\ NO \end{array} \right\} :: \left\{ \begin{array}{c} Ny-EA \\ OR \end{array} \right\} . \left\{ \begin{array}{c} ay-aE \\ AO. \end{array} \right\}$ Fig. 2.

Vel ſic: Si ad Axem Parabolæ (*ay*) ordinatim applicata recta (*NR*,) ſecet aliam Diametrum (ut *Ao*, productam, ſi opus fuerit) in duo Segmenta (*NO*, *OR*;) Dico,

Rectan-

9 11 $\frac{Ny^2}{L} = ay$

9 12 $\frac{Ae^2}{L} = ae$

11 ∽ 12 13 $\left\{\begin{matrix} ay \backsim ae \\ \text{u}, AO \end{matrix}\right\} = \frac{Ny^2 \backsim Ae^2}{L} = \left\{\begin{matrix} \frac{\overline{Ny + Ae} \times \overline{Ny - Ae}}{L} \\ \text{u}, \frac{NO \times OR}{L}. \end{matrix}\right\}$ Fig. 2.

14 14 g $\left\{\begin{matrix} L \,.\, NO :: OR \,.\, AO \\ L \,.\, no :: oR \,.\, Ao. \end{matrix}\right\}$ Which is its second special property.

The general property of a Parabole (§ 7) is expressed in words, thus:

The Squares of the Right Lines (*No*, *no*,) ordinately applied to the Axe (or Diameter), to their distances from the Vertex (*A*): Or (which is all one), The Squares of the Ordinates (*No*, *no*,) to their Abscists (*AO*, *Ao*,) are directly proportional.

1. The first special property of a Parabole; is expressed in words, thus: (§ 10.)

A Right Line (as *NO*) ordinately applied to the Axe, is a Mean Proportional, between its Abscist (*AO*,) and the *Latus Rectum* (*L.*) Fig. 1.

Or, *As* the Abscist (*AO*,) *Is* to the Ordinate, of which it is the Abscist (*NO*:) *So* is the same Ordinate (*NO*,) *To* the *Latus Rectum* (*L.*) *viz.* $\left\{\begin{matrix} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao \end{matrix}\right\}$

2. But the second special property (§ 14.) thus:

If to the Axe of a Parabole (*ay*,) be ordinately applied two Right Lines (*Nq*, *AE*;) I say,

That the *Latus Rectum*, is to the sum of those two Right Lines; as their difference, is to the difference of their Abscists; *viz.* $L \,.\, \left\{\begin{matrix} Ny + EA \\ NO \end{matrix}\right\} :: \left\{\begin{matrix} Ny - EA \\ OR \end{matrix}\right\} . \left\{\begin{matrix} ay - aE \\ Ao \end{matrix}\right\}$ Fig. 2.

Or thus: If any Ordinate (as *NR*) applied to the Axe of a Parabole, cut any other Diameter (as *Ao*, produced if need be) into two Segments, (as *NO*, *OR*;) I say,

The

Rectangulum ſub latere Recto, & Diametro intercepta, eſſe æquale Rectangulo ſub Segmentis. Vel,

Ut Latus Rectum, ad unum Segmentorum; ſic alterum, ad Diametrum interceptam.

4. Atque hinc (per modum Conſectarii) eluceſcet modus Parabolam in plano conſtruendi, ex datis ejus Axe A O, & latere ejus Recto L.

1. Ad Rectam A O, applicentur ad Angulos Rectos infinitæ Parallelæ (ut h N N h, &c.)

Menſurata A I = L; & in Axe A O, dato quocunque puncto (verbi gratiâ), O ſumpto, reperiatur inter duas Rectas (A I, A O,) media Proportionalis A L, applicanda Axi in Puncto O, (h. e. O N, vel B N =) A L; & ſic de aliis infinitis hujuſmodi Rectis modo prædicto reperiendis & conſtituendis, ex diverſis punctis Axis A O: Linea curva incedens per extrema dictarum parallelarum (verbi gratiâ inſtar omnium) per extremum N Rectæ O N (vel B N,) delineabit Parabolam. *Fig.* 3.

2. Qui quidem Parabolam deſcribendi modus ſatis admodùm facilis ſit licet, expeditior tamen mihi videtur ille, qui a Triangulo Rectangulo Iſoſcele, (ſicuti Hyperbole abs obtuſangulo, & Ellipſis ab acutangulo Triangulo Iſoſcele derivata,) originem ſuam trahit. *Fig.* 3.

Exponatur itaque Triangulum Iſoſceles (b a c), rectangulam ad (a,) cujus Latera (a b, a c) ſint æqualia: In perpendiculari (a o) demiſſâ, ſumatur a f = $\frac{L}{2}$, & biſecetur (a f) in A, (quod erit Vertex Parabolæ:) Ductis infinitis Rectis (h o h, &c.) Baſi (b c) parallelis; abs f, tanquam à Centro, intervallo verò infinitarum ipſarum Parallelarum (exempli gratiâ, inſtar omnium) intervallo (O h, vel) B h, deſcribatur Arcus ſecans ipſam Parallelam (B h,) in puncto N; (hoc eſt, ſtatuenda eſt f N = B h:) Et ſic abs f, infinitis verò hujuſmodi intervallis, deſcribantur infiniti alii Arcus, ipſas proprias Parallelas ſecantes in (N). Dico, Lineam curvam incedentem per omnes illas Interſectiones ad N, delineare Parabolam. *Fig.* 3.

1

*Demonſtr.*

The Rectangle made of the *Latus Rectum*, and intercepted Diameter, is equal to the Rectangle made of both the Segments.

Or, *As* the *Latus Rectum*, *Is* to one of the Segments: *So* is the other, *To* the intercepted Diameter.

4. And hence (by way of Consectary) may be found out a way, how to describe a Parabole in Plano; having the Axe A o, and *Latus Rectum* (L) given.

1. To the Axe A O, let be applied to Right Angles infinite Parallels (as h N N h, *&c.*)

Make A I = L; and any Point (as o) being taken, in the given Axe A O, find out between the two Right Lines (A I, A O,) a mean Proportional (A L,) to be applied to the Axe, in the Point O, (*i. e.* O N, or B N) = A L; and so of infinite other Right Lines of this sort, to be found after the same manner, and to be placed from divers points of the Axe A O. A crooked Line passing through the Extreams of the said Parallels (for Example, one for all) through the Extream (N), of the Right Line O N (or B N), will describe a Parabole. *Fig.* 3.

2. Although this way of describing a Parabole, is easie enough; yet that way seems to me to be more expedite, which hath its origin from a rectangular Isosceles Triangle, (as an Hyperbole from an obtusangular, and an Ellipse from an acutangular Isosceles Triangle.) *Fig.* 3.

Let therefore an Isosceles Triangle (a b c) rectangular at (a) be made, whose Sides (ab, ac) are equal: In the Perpendicular (a o) let fal'n on the Base, let be taken a f = $\frac{1}{2}$; which being bisected in (A), will be the Vertex of the Parabole. Infinite Right Lines (as h o h, *&c.*) being drawn parallel to the Base (b c,) from f, (as from a Center) but at the distance of those infinite Parallels, (for Example, one for all) at the distance of (O h, or) B h, let an Arch be described, cutting the said Parallel (B h) in the Point N, (*i. e.* making f N = B h, or a B:) And so from f, at infinite other Distances of this sort, infinite other Arches must be described, cutting their proper Parallels in (N:) I say, A crooked Line (as N A N) passing through all those Intersections, will describe a Parabole. *Fig.* 3.

*Demonstr.*

1 — 2 $af = \frac{L}{2}$. $aA = Af = \frac{L}{4}$; g°, $4\,Af = L$.

Constr. — 3 $(aA + AB =)\ Af + AB = (aB = Bh =)\ fN$.

⊙ — 4 $Af^2 + 2Af \times AB + AB^2 = fN^2$.

· — 5 $AB \backsim Af = fB$.

⊙ — 6 $Af^2 - 2Af \times AB + AB^2 = fB^2$.

47, è 1. — 7 $(BN^2, u)\ NO^2 = fN^2 - fB^2 = (4-6)\ 4Af \times AB = (\S\,2, L \times AB, u)\ L \times AO$: hoc est,

7 — 8 $L . NO :: NO . AO$; quæ est prima propr. special.

## CAP. II.

***De Equationibus omnibus quartum gradum non excedentibus, quomodolibet affectis, construendis, & ipsarum radicibus tam falsis quam veris reperiendis.***

QUO quidem artificio, Parabolâ suppositâ descriptâ, punctum reperiatur, à quo (tanquam à centro) intervallo quodam determinando, circulus possit describi, qui ita secet vel tangat Parabolam, ut à punctis concursûs rectæ eductæ omnes omnium Æquationum quartum gradum non excedentium, quomodolibet affectarum, radices tam falsas quàm veras determinent; ut hujus negotii præcipuus est cardo, & unicum illud maximè inquirendum; ita quod ad amussim præstat Regula hæc subnexa; quam, distinctionis ergò, liceat appellare *Centralem*, vel *Locum*.

*Regula Centralis.*

Pars 1 $\frac{L}{2} + \frac{p^2}{8L} \overset{+}{\backsim} \frac{q}{2L} = b = AD$.

Pars 2 $\frac{p}{4} + \frac{p^3}{16L^2} \overset{+}{\backsim} \frac{pq}{4L^2} \overset{+}{\backsim} \frac{r}{2L^2} = d = DH$.

In

*Demonstr.*

1 — 2 $af = \frac{L}{2}.\ aA = Af = \frac{L}{4}$; g°, $4Af = L$.

*Constr.* — 3 $(aA + AB =)\ Af + AB = (aB = Bh =)\ fN$.

⊙ — 4 $Af^2 + 2Af \times AB + AB^2 = fN^2$.

5 $AB \backsim Af = fB$.

⊙ — 6 $Af^2 - 2Af \times AB + AB^2 = fB^2$.

47, è 1. — 7 $(BN^2, u)\ NO^2 = fN^2 - fB^2 = (4 - 6)\ 4Af \times AB = (\S\,2, L \times AB, u)\ L \times AO$: that is,

7 — 8 $L . NO :: NO . AO$; which is the first ſpecial property of a Parabole.

## CHAP. II.

### *Of the Conſtruction of all Equations, not exceeding the fourth Degree, howſoever affected; as alſo the finding all their Roots, as well falſe as true.*

By what artifice, a Parabole being ſuppoſed to be deſcribed, a certain Point may be found, from which, (as from a Center) at a certain diſtance to be determined) a Circle may be deſcribed, which may ſo cut or touch a Parabole, that from the Points of their Meeting, Right Lines drawn may determine all the Roots, as well falſe as true, of all Equations, not exceeding the fourth Degree, howſoever affected: As it is the Hinge on which all this buſineſs hangs, and the only thing chiefly to be enquired after; ſo is it that, which this following Rule exactly performs, which for diſtinction-ſake we may call the *Central Rule*, or *Place*.

*The Central Rule.*

$$\text{Part}\left\{\begin{array}{ll} 1 & \frac{L}{2} + \frac{p^2}{8L} \underset{\backsim}{+} \frac{q}{2L} = b = AD. \\ 2 & \frac{p}{4} + \frac{p^3}{16L^2} \underset{\backsim}{+} \frac{pq}{4L^2} \underset{\backsim}{+} \frac{r}{2L^2} = d = DH. \end{array}\right.$$

In quâ est observanda occurrit, $\left\{\begin{matrix}\text{Quantitatum}\\ \text{Signorum}\end{matrix}\right\}$ determinatio.

1. Si qua Quantitatum ( p, q, r,) in Æquatione propositâ deficeret, quæ deficit, à Regulâ Centrali (ut necesse est) est abdicanda, & à reliquis determinanda est Regula.

2. Quod ad Signa determinanda spectat, notandum;

1. In Regulâ continuò habebitur $+\frac{r}{2L^2}$, nisi quum in Æquatione propositâ, p & r diversis signis affici contigerit; quo in casu, Signo negativo ( nempè $-\frac{r}{2L^2}$) mulctari oportet.

2. Quocunque signo, in Æquatione propositâ, denotari acciderit Quantitas (q;) contrario quidem in Regulâ (aliâ implicata licet) designanda est.

Non quidem est hujus loci, nec tanti momenti, ut moneam, oportere aliquando fieri, inter demonstrandum, (licet nunquam inter construendum) Signorum Enallagen; nempè, quum excessus in Regulâ fuerit penes quantitates Signo Negativo adfectas; quum cuique figuram inspicienti facilè videre est, an punctum D, citra vel ultra verticem Axis vel Diametri; vel punctum (H) ad dextram vel sinistram ejusdem Axis vel Diametri cadere contigerit, necne.

Missâ autem hâc (forsan subobscurâ) verborum ambage, Regulam centralem cuilibet Æquationum Classi propriam, brevi Synopsi oculis subjiciam. Vide Synopsin.

Hisce benè perspectis, & probè intellectis, ad altiora & penitiora hujusmodi mysteria facilior erit aditus.

In which is to be observed, The determination of the $\left\{ \begin{array}{l} \text{Quantities.} \\ \text{Signs.} \end{array} \right.$

1. If any one of the Quantities (p, q, r,) be wanting in the Æquation proposed, that which wants, must (of necessity) be excluded the Central Rule; and the true Rule is to be determined by the remaining Quantities only.

2. For the determination of the Signs;

1. In the Rule must always be had $+\frac{r}{2L^2}$, unless in the Equation proposed, it happens, that p and r are affected with divers Signs; in which case, it must be $-\frac{r}{2L^2}$.

2. With what Sign soever the Quantity (q) is noted in the Equation proposed, it must be marked with its contrary Sign (although involved with another Quantity) in the Rule.

It pertains not to this place, neither is it of such moment to notifie, That sometimes, whiles demonstrating (although never whiles working) a change of the Signs must be made; *viz.* When in the Rule the Negative Quantities happen to exceed the Affirmatives, seeing any one that inspects the Figure may easily discover, whether or no the Point D happens to fall below or above the Vertex of the Axe or Diameter; or the Point (H) on the right or left side of the said Axe or Diameter.

But passing by this (perhaps obscure) way of Discourse, we will present, in a brief Synopsis, the prospect of each Central Rule proper to each Class of Equations. See the Synopsis.

These things clearly perceived, and rightly understood, a more easie entrance will be had, to the more hidden and higher Mysteries of this kind.

## *Regulæ Centralis ad Parabolam applicatio; sive* Regula Generalis.

DEscribatur Parabola (N A M,) cujus Latus Rectum sit L (sive 1), Axisque (a y, vel) A y; ad quem, si in Æquatione habeatur p, ordinatìm ap-
1 plicetur $BA = \frac{p}{2}$, occurrens Parabolæ in B & A; & (ab alterutro, puta) ex A, ducatur recta Parallela Axi AO: Si verò in Æquatione deficeret p, nulla erit necessitas, vel applicandi BA ad Axem; vel ducendi Diametrum A y.

Tum in $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ A y, si in Æquatione, p
2 $\left\{\begin{matrix}\text{abfuerit}\\ \text{deficerit}\end{matrix}\right\}$ sumatur $AD = b$ (suprà inventæ, Æquationi propositæ congruæ); cujus quælibet quantitas Signo + adfecta, vel denotata), aggregatìm vel singulatìm in $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ deorsum, versus y est disponenda, & exinde, Quantitas negativa (si qua fuerit,) in eodem $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ (continuato, si opus fuerit,) sursum versus A est collocanda; inventumque erit punctum D.

A quo Puncto (nempè D,) erigatur perpendicula-
3 ris ad A y, Recta $DH = d$ (suprà inventæ, Æquationi propositæ congruæ); cujus etiam quælibet quan-
4 titas Signo + denotata, aggregatìm vel singulatìm, versus sinistram est in ipsâ perpendiculari disponenda, & exinde, quælibet reliqua quantitas (si qua fuerit) Signo (—) designata, est in ipsâ (continuatâ, si
5 opus fuerit,) versus dextram collocanda; inventumque erit (punctum, vel) Circuli centrum (H.)

Quo invento, & connexâ H A, oportet ex Cen-
6 tro (H) circulum (H A M) describere, cujus Semidiameter

## *The application of the Central Rule to a Parabole*; *or*, The General Rule.

LET a Parabole NAM be described, whose *Latus*
*Rectum* is L (or 1), and Axe (a y, or) A y;
to which, if in the Equation be found p, let there be
1 ordinately applied $BA = \frac{p}{2}$, meeting the Parabole in
B and A; and from (either of which, suppose) A,
let there be drawn parallel to the Axe, the Right Line
AO: But if p be wanting in the Equation, there will
be no need either of applying BA to the Axe, or of
drawing the Diameter AO.

Then in the $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$ AO, if in the Equation p
2 be $\left\{\begin{matrix}\text{had, or}\\ \text{wanting}\end{matrix}\right\}$, make $AD = b$ (before found, proper
to the Equation proposed); all whose Quantities noted
with the Sign +, aggregately or severally are to be
disposed on the $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$ downwards towards y;
and from thence, the Quantity (if any) noted with
a Negative Sign (—), is to be placed upwards to-
wards A, on the same $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$; and the Point D
will be found.

From which Point (*viz.* D) let be erected per-
3 pendicular to A y, the Right Line $DH = d$ (above
found, proper to the Equation proposed); all whose
4 Quantities also, marked with the Sign +, are aggre-
gately or severally in the said Perpendicular, to be
disposed towards the left hand; and from thence, those
remaining Quantities (if any,) marked with the Sign
5 (—), are to be placed on it (continued, if need be)
towards the right hand, and the (Point, or) cen-
ter of the Circle (H) will be found.

Which being found, and HA connected, from the
6 Center (H) must be described a Circle (HAM,)
whose

diameter sit HA, si Equatio, non sit Biquadratica,
hoc est, si non habeatur Quantitas S.

7 Quod si habeatur S, & Signo quidem negativo ad-
fecta (nempe — S), oportet ulteriùs in hâc Lineâ
8 AH, utrinque productâ, ex unâ parte sumere $AI = L$
9 (sive 1), & ex alterâ parte $AK = \frac{S}{L^3}$; descripto-
10 que Semicirculo, cujus Diameter (IK,) erigere AL
perpendicularem ad AH, quæ occurrat huic Semicir-
culo (ILK,) in Puncto L; quod illud ipsum est, per
quod alter Circulus (NLM) transire debet.

11 Quod si verò habeatur + S, oportet insuper in alio
Semicirculo, cujus Semidiameter est AH, inscribere
$AZ = AL$ inventæ; inventumque erit Punctum Z,
per quod primus Circulus quæsitus transire debet.

Circulus igitur descriptus transiens per A, si defe-
cerit S (ut suprà § 6.); vel si habeatur S; transiens
12 per L, si sit — S (ut suprà § 10.); per Z verò, si
sit + S, (ut suprà § 11.) secare vel tangere possit
Parabolam in 1, 2, 3, aut 4 punctis; à quibus, si ad
Axem vel Diametrum demittantur perpendiculares,
obtinebuntur omnes Æquationis radices, tam falsæ,
quàm veræ; nimirum,

13 1. Si quidem in Æquatione defecerit p, & sit — r;
veræ radices erunt illæ harum Perpendicularium, quæ
ad sinistram partem Axis reperientur (ut NO); &
reliquæ (ut MO) erunt falsæ.

14 2. Si verò in Æquatione habeatur p, & sit — p; veræ
radices erunt illæ, quæ ad sinistram partem Axis (ut
NO); falsæ verò (ut MO) quæ ad dextram repe-
rientur.

15 Sed contrà; si sit + p; veræ quidem cadent ad dex-
tram partem Axis vel Diametri (ut MO); falsæ
verò (ut NO), ad sinistram.

Notandum: Si hic Circulus neque secat, neque tan-
git Parabolam in aliquo puncto; indicio est, impos-
sibilem esse Æquationem, nullamque admittere radi-
cem, sive veram sive falsam, sed tantum imagi-
narias.

Si

whose Semidiameter H A, if it be not a Biquadratic Equation, (*i. e.*) if the Quantity S be wanting.

7 But if S be had, and it be — S, then further in this Line A H, both ways produced, must be taken

8 on the one side A I = L (or 1), and on the other

9 side $AK = \frac{S}{L^3}$; and a Semicircle being described,

10 whose Diameter I K must be erected A L perpendicular to A H, which may meet this Semicircle (I L K,) in the Point L; which is that very Point, through which the other Circle (N L M) must pass.

11 But and if be had + S, there must moreover in another Semicircle, whose Diameter is A H, be inscribed A Z = A L found; and the Point Z will be found, by which the first Circle sought ought to pass.

A Circle therefore described passing through A, if S be wanting (as before, § 6.); or if S be had, passing

12 through L, if it be — S (as above, § 10.); but through Z, if it be + S, (as above, § 11.) may cut or touch the Parabole in 1, 2, 3, or 4 Points; from which, if Perpendiculars be demitted to the Axe or Diameter, all the Roots, as well false as true, will be had; *viz.*

13 1. If in the Equation p be wanting, and it be — r; those of these Perpendiculars will be the true Roots, which shall be found on the left side of the Axe (as N O); and the rest (as M O) false.

14 2. But if in the Equation p be had, and it be — p; those will be the true Roots, which shall be found on the left side of the Axe (as N O); and those false (as M O), which on the right.

15 But contrariwise; if it be + p, those will be the true Roots, which shall fall on the right side of the Axe or Diameter (as M O); and those false (as N O), which on the left.

*Note:* If this Circle neither cuts nor touches the Parabole in any Point, it is a token of an impossible Equation, and that it admits of no Roots, whether true or false, but only imaginary ones.

Si sumatur *Latus Rectum* L = 1 ( sive Unitati ) dico L, cum omnibus suis gradibus omitti posse, quod semel annotâsse sufficiat.

Quorum omnium demonstratio, singulas formulas Æquationum percurrendo, in sequentibus innotescet. Priusquam verò rem aggressus fuero, liceat mihi paulisper ἀπολογίζειν;

Non est, quòd Parabolam ( ut veteres olim *Junonem Lucinam* ), ad Classis primi, secundi, & partis prioris septimi, nedum prioris partis quinti partum, in auxilium invocemus; cùm faciliore forsan nixu, famulante solo circulo, unico, partu satis admodum maturè levetur: Quàm dexterè autem & auspicatò obstetricis ( ancillante solo Circulo ) partes hâc in re possit agere ( mysteriis quidem gravida ) Parabola; non abs re, imo operæ forsan erit pretium, ( ut Regulæ generalis Amplitudinem ostendamus, ) breviter perstringere.

## CLAS. I.

*De Æquationibus quartæ Dimensionis construendis, ubi omnes termini ( p, q, r, ) deficiunt; vel, ubi affectis sub nullo gradu Paradoxo.*

OMnes hujus census Æquationes ad unicam quidem solam formulam sunt reducibiles.

1. $x^4 * * * - S = 0$.

Synops. Cl. 1. *Regula Generalis.* $\frac{L}{2} = b = AD. \quad 0 = d = DH.$

Reg. Gen. Describatur Parabola ( N A M ), cujus Latus Rectum sit L ( sive 1 ), Axique A y, in quo sumatur

2 1 $AD = AH = b = \frac{L}{2}$. Ex unâ parte A H ( utrinque

8 2 productæ ) sumatur $AI = \frac{L}{2}$, & ex alterâ parte,

AK

If be taken the *Latus Rectum* $L = 1$ (or an Unity), I say, L with all its degrees may be omitted; which once to have noted may suffice.

The Demonstration of all which, running through each particular form of Equations, will appear in our following Discourse. Which before I shall attempt, I shall take leave to make this Apology.

Though no necessity of invoking a Parabole (as of old they did *Juno Lucina*), to midwife forth the two first Classes of Equations, as also the former part of the seventh, much less the former part of the fifth: Seeing without her assistance a Circle only may with more ease perhaps, and timely enough bring it to birth; yet how dextrously and luckily a Parabole (big with Mysteries) can in this business act the Midwife's part, by help of a Circle only, as it will not be altogether besides our purpose, so perhaps worth our while to shew, sith that it blazons the Amplitude of our General Rule.

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## CLAS. I.

*Of the Construction of Equations of the fourth Dimension, where all the terms (p, q, r,) are wanting, or where affected under no Parodic Degree.*

ALL Equations of this kind are reducible to this one only form.

1. $x^4 * * * + S = 0$.

*Central Rule.* $\frac{L}{2} = b = AD$. $0 = d = DH$.

*Gen. Rule.* Let a Parabole (NAM) be described, whose *Latus Rectum* L (or 1), and Axe Ay; in which take

2 1 $AD = AH = b = \frac{L}{2}$. On the one part of AH (both *Fig.* 4.

8 2 ways produced) let be made $AY = L$; and on the other

| | | |
|---|---|---|
| 9 | 3 | $AK = \frac{S}{L^3}$; descriptoque Semicirculo, cujus Diameter |
| 10 | 4 | sit IK, erigenda est AL ad Axem perpendicularis, quæ occurrat huic Semicirculo (ILK), in puncto L. Centro quidem H, intervallo verò HL, describatur Circulus (NLM), qui secabit Parabolam in punctis N & M: A quibus demissæ rectæ (NO, MO), erunt radices quæsitæ; quarum altera vera, altera falsa. |
| | | *Formula.* $x^4 * * * - S = 0$. |

*Demonstrat.*

Fig 4.

| | | |
|---|---|---|
| 1. | 5 | AD, vel $AH = b = \frac{L}{2}$. |
| 3 × 4 | 6 | $AI \times AK = AL^2$; ob circulum:<br>$(L \times \frac{S}{L^3} =)\ AL^2 = \frac{S}{L^2}$. |
| 47, è 1.<br>Q. 5. + Q. 6. | 7 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad.<br>$b^2 + \frac{S}{L^2} =$ Q. Rad. |
| Supp. | 8 | $NO = x$. |
| Supp. | 8 | $MO = -x$. |
| Ob para. | 9 | L . NO :: NO . AO.<br>$L \,.\, x :: x \,.\, \frac{x^2}{L} = AO$. |
| Ob para. | 9 | L . MO :: MO . AO.<br>$L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO$. |
| 9 ∽ 5. | 10 | AO ∽ AD = DO.<br>$\frac{x^2}{L}\ (-b, u) - \frac{L}{2} = DO$. |
| [illegible] | 11 | $b^2 + \frac{x^4}{L^2} - x^2 = DO^2$. |
| Q. 8. | 12 | $x^2 = NO^2$. |
| Q. 8. | 12 | $x^2 = MO^2$. |

DO²

9 3 other side AK $= \frac{S}{L^3}$: And a Semicircle being descri-
10 4 bed, whose Diameter IK, erect AL perpendicular to the Axe, which may meet this Semicircle (ILK) in the Point L. Center H, and distance HL, let a Circle (NLM) be described, which will cut the Parabole in the Points N and M: From which Perpendiculars demitted to the Axe (as NO, MO,) will be the Roots desired, whereof the one true, the other false.

*Form.* $x^4 * * * - S = 0.$

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47, è 1. / 11 + 12 — 13 $\begin{cases} DO^2 + NO^2 = (DN^2 =)\ Q.\,Rad. \\ b^2 + \frac{x^4}{L^2} = Q.\,Rad. \end{cases}$

47, è 1. / 11 + 12 — 13 $\begin{cases} DO^2 + MO^2 = (DM^2 =)\ Q.\,Rad. \\ b^2 + \frac{x^4}{L^2} = Q.\,Rad. \end{cases}$

13 = 7 — 14 $\frac{x^4}{L^2} = \frac{S}{L^2}$; in $L^2$.

$\times L^2$ — 15 $x^4 = S.$

*Transp.* — 16 $x^4 * * * - S = 0.$ *Q. e. d.*

*Illustrat.* *Central.*

$\begin{cases} x^4 * * * - 6561 = 0 \\ x^4 * * * - 0.6561 = 0 \end{cases}$ $\frac{L}{2} = 0.5 = b.$ *Fig. 4.*

$\begin{cases} NO = x = 9 \\ MO = -x = -9 \end{cases}$

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CLAS.

# CLAS. II.

*De Æquationibus, 1. Secundæ Dimensionis construendis, ubi deficit secundus Terminus, nempe p: 2. Quartæ Dimensionis, ubi deficit secundus & quartus Terminus, (nempe p & r;) vel affectis tantum sub Quadrato, vel secundo Gradu Parodico.*

OMnes Æquationes $\left\{\begin{matrix}\text{secundæ}\\ \text{quartæ}\end{matrix}\right\}$ Dimensionis, ad $\left\{\begin{matrix}1\\ 3\end{matrix}\right\}$ formulas possint reduci.

$$\left\{\begin{matrix}1.\ x^2 * -q=0 \\ 2.\ x^2 * +q=0 \\ \text{impossibilis.}\end{matrix}\right. \quad \begin{matrix}\left\{\begin{matrix}1\\ 3\end{matrix}\right\}\left\{\begin{matrix}x^4 * -qx^2 * -S=0\\ x^4 * -qx^2 * +S=0\end{matrix}\right\} \\ \left\{2.\ x^4 * +qx^2 * -S=0.\right.\end{matrix}$$

*Regula Centralis.*

*Synops. Cl. 2.* Si $\left\{\begin{matrix}-q\\ +q\end{matrix}\right\} \frac{L}{2} \begin{matrix}+\\ \backsim\end{matrix} \frac{q}{2L} = b = AD. \quad o = d = DH.$

*Reg. Gen*

Suppositâ igitur descriptâ Parabolâ (NAM), cujus Latus Rectum sit L (seu 1), Axisque A y; opor-

1 1 tet facere $AB = \frac{L}{2}$ continuò; tum sumptâ b D, vel

2 2 $bH = \frac{q}{2L}$; collocetur in Axe ulteriùs versus y, si in Æquatione habeatur $-q$; sed sursum, in Axe (continuato, si opus fuerit) si habeatur $+q$.

Tum ex Centro H, oportet Circulum describere *Fig. 5.*

6 3 (NAM) cujus Semidiameter sit HA, si in Æquatione non habeatur Quantitas S.

7 4 Ast si habeatur S; & sit $-S$; oportet ulterius in hac lineâ AH (productâ utrinque) ex unâ parte su-

8 5 mere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$; describ- *Fi. 6, 7, 8, 9*

9 6 toque

# CLAS. II.

*Of the Construction of Equations, 1. Of the second Dimension, where the second Term (p) is wanting. 2. Of the fourth Dimension, where the second and fourth Term (viz. p and r) are wanting; or affected only under a Square, or the second Parodic Degree.*

ALL Equations of the $\left\{\begin{matrix}\text{second}\\ \text{fourth}\end{matrix}\right\}$ Dimension, are reducible to these $\left\{\begin{matrix}1\\ 3\end{matrix}\right\}$ forms.

$$\left\{\begin{matrix} 1.\ x^2 * - q = 0 \\ 2.\ x^2 * + q = 0 \\ \text{impossible.} \end{matrix}\right. \quad \begin{matrix} \left\{\begin{matrix}1\\3\end{matrix}\right\}\left\{\begin{matrix} x^4 * - qx^2 * - S = 0 \\ x^4 * - qx^2 * + S = 0 \end{matrix}\right\} \\ 2.\ x^4 * + qx^2 * - S = 0. \end{matrix}$$

*Central Rule.*

*Synops. Cl.* 1. If $\left\{\begin{matrix}-q\\+q\end{matrix}\right\} \frac{L}{2} \pm \frac{q}{2L} = b = AD. \quad 0 = d = DH.$

*Gen. Rule* A Parabole being therefore supposed to be described (as N A M), whose *Latus Rectum* L (or 1), and

1 1 Axe A y; make always $Ab = \frac{L}{2}$; then taking B D,

2 2 or $bH = \frac{q}{2L}$, let it be placed (from b,) farther downwards towards y, if in the Equation be had $-q$; but upwards on the Axe (continued, if need be) if in the Equation be had $+q$.

6 3 Then from the Center H, let a Circle (N A M) be described, whose Semidiameter H A, if in the Equation be not had the Quantity S. *Fig.* 5.

7 4 But if S be had, and it be $-S$, then must be taken farther in this Line A H (both ways produced), on

8 5 6 the one side $AI = L$, and on the other $AK = \frac{S}{L^3}$; and

9 a Semi-

toque Semicirculo (ILK), cujus Diameter IK, eri-
10 7 gere AL perpendicularem AH, quæ occurrat huic Semicirculo (ILK), in puncto L.

11 8 Quòd si verò habeatur + S; oportet insuper in alio Semicirculo, cujus Diameter sit AH, inscribere $AZ = AL$ inventæ. *Fig.* 7.

9 Circulus igitur descriptus, transiens per A, si defecerit S (ut suprà § 3.) vel si habeatur S, transiens per L, si sit − S (ut suprà § 7.); per Z verò si sit + S, (ut suprà § 9.) secabit vel tanget Parabolam in 2 aut 4 punctis; à quibus si ad Axem demittantur perpendiculares, omnes radices tam falsæ, quàm veræ obtinebuntur; quarum veræ ex unâ, falsæ verò ex alterâ parte Axis contingerint.

*Imposs.*

$$\left\{\begin{array}{l} 1.\ x^2 * - q = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 * - S = 0 \\ x^4 * - qx^2 * + S = 0 \end{array} \\ 2.\ x^2 * + q = 0 \left\{\begin{array}{l} 2 \\ \phantom{2} \end{array}\right\} \begin{array}{l} x^4 * + qx^2 * - S = 0 \\ * * \quad * * \quad * \end{array} \end{array}\right.$$

*Cas.* 1. Ubi − q. *Fi.* 5,6,7

*Demonstrat.*

1 + 2 10 $\left\{\begin{array}{l} Ab + (bD,\ u)\ bH = (AD,\ u)\ AH = b. \\ \frac{L}{2} + \frac{q}{2L} = (AD,\ u)\ AH = b. \end{array}\right.$

⊙ 11 $b^2 = (AD^2,\ u)\ AH^2 = Q.$ Rad. in quadratici. *Fig.* 5.

5 × 6 12 $\left\{\begin{array}{l} AI \times AK = (\text{ob circl.})\ AL^2 = (\text{p. constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, è 1; 11 + 12 13 $\left\{\begin{array}{l} AH^2 + AL^2 = (HL^2 =)\ Q.\ \text{Rad.} \\ b^2 + \frac{S}{L^2} = Q.\ \text{Rad.} \end{array}\right.$ *Fig.* 6.

47, è 1; 11 − 12 14 $\left\{\begin{array}{l} AH^2 - AZ^2 = (HZ^2 =)\ Q.\ \text{Rad.} \\ b^2 - \frac{S}{L^2} = Q.\ \text{Rad.} \end{array}\right.$ *Fig.* 7.

NO

a Semicircle (ILK), whose Diameter IK being
10 | 7 | described, must be erected AL perpendicular to AH, which may meet this Semicircle (ILK), in the Point L.

11 | 8 | But if be had $+S$; then moreover in another Semicircle, whose Diameter AH, must be inscribed AZ = AL found. *Fig.* 7.

9 | A Circle therefore described passing through A, if S be wanting, (as § 3.) or if S be had, passing through L, if it be $-S$ (as § 7.); but through Z, if it be $+S$, (as § 9.) will cut or touch the Parabole in 2 or 4 Points; from which, if Perpendiculars be demitted to the Axe, all the Roots, as well false as true, will be had; of which, those true on the one part, on the other part false.

*Imposs.*

$$\left\{\begin{array}{ll} 1.\ x^2 * - q = 0 & \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 * - S = 0 \\ x^4 * - qx^2 * + S = 0 \end{array} \\ 2.\ x^2 * + q = 0 & \left\{\begin{array}{l} ? \\ 2 \end{array}\right\} \begin{array}{l} x^4 * + qx^2 * - S = 0 \\ * * \quad * * \quad * \end{array} \end{array}\right.$$

---

*Supp.* 15 | $NO = x$.

*Supp.* 15 | $MO = -x$.

*Ob para.* 16 | $\left\{\begin{array}{l} L . NO :: NO . AO. \\ L . x :: x . \frac{x^2}{L} = AO. \end{array}\right.$

*Ob para.* 16 | $\left\{\begin{array}{l} L . MO :: MO . AO. \\ L . -x :: -x . \frac{x^2}{L} = AO. \end{array}\right.$

16 ∽ 10 | 17 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HO. \\ \frac{x^2}{L}(-b, u) - \frac{L}{2} - \frac{q}{2L} = (DO, u)\ HO. \end{array}\right.$

⊖ | 18 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = (DO^2, u)\ HO^2$.

*Q.* 15. | 19 | $x^2 = NO^2$.

*Q.* 15. | 19 | $x^2 = MO^2$.

| | | | |
|---|---|---|---|
| 47, *e* 1.<br>18 + 19 | 20 | $\left\{\begin{array}{l} DO^2 + NO^2 = HN^2. \\ b^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} = HN^2 = Q.\ Rad. \end{array}\right.$ | |
| 47, *e* 1.<br>18 + 19 | 20 | $\left\{\begin{array}{l} DO^2 + MO^2 = HM^2. \\ b^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} = HM^2 = Q.\ Rad. \end{array}\right.$ | |
| 20 = 11 | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 22 | $x^2 * - q = 0$. *Q. e. d.* in Quadratic. | *Fig.* 5. |
| 20 = 13 | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4 - qx^2 = S$. | |
| *Transp.* | 25 | $x^4 - qx^2 - S = 0$. *Q. e. d.* in Biquadr. | *Fig.* 6. |
| 20 = 14 | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4 - qx^2 = -S$. | |
| *Transp.* | 28 | $x^4 - qx^2 + S = 0$. *Q. e. d.* in Biquadr. | *Fig.* 7. |

*Illustrat.*

*q.* *Central.*

$$\left\{\begin{array}{l} x^2 * - 144 = 0 \\ x^2 * - 1.44 = 0 \end{array}\right\} \qquad \frac{L}{2} = 0.5$$

$$\left\{\begin{array}{l} NO = x = 12 \\ MO = -x = -12 \end{array}\right\} \qquad + \frac{q}{2L} = 0.72$$

$$b = 1.22 = AD.$$

*Fig.* 5.

*q.* *s.* *Central.*

$$\left\{\begin{array}{l} x^4 * - 108\,x^2 * - 5184 = 0 \\ x^4 * - 1.08\,x^2 * - 0.5184 = 0 \end{array}\right\} \qquad \frac{L}{2} = 0.5$$

$$\left\{\begin{array}{l} NO = x = 12 \\ MO = -x = -12 \end{array}\right\} \qquad + \frac{q}{L\,2} = 0.54$$

$$b = 1.04 = AD.$$

*Fig.* 6.

$x^4$

q. s.

$$\begin{cases} x^4 * - 292x^2 * + 9216 = 0 \\ x^4 * - 2.92x^2 * + 0.9216 = 0 \end{cases}$$

$$\begin{cases} NO = x = 16. & no = x = 6 \\ MO = -x = -16. & mo = -x = -6 \end{cases}$$

*Central.* Fig.7.

$$\frac{L}{2} = 0.5$$

$$+\frac{q}{2L} = 1.46$$

$$b = 1.96 = AD.$$

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*Cas.* 2. Ubi $+ q$.

Ubi $\begin{cases} 1. \ \frac{L}{2} \sqsupset \frac{q}{2L} \\ 2. \ \frac{q}{2L} \sqsupset \frac{L}{2} \end{cases}$ Fig.8, 9.

*Demonstrat.*

1 — 2 10 $\begin{cases} Ab - (bD, u)\ bH = (AD, u)\ AH. \\ \frac{L}{2} - \frac{q}{2L} = (AD, u)\ AH = b. \end{cases}$ Fig.8.

2 — 1 10 $\begin{cases} (bD, u)\ bH - Ab = (AD, u)\ AH. \\ \frac{q}{2L} - \frac{L}{2} = (AD, u)\ AH = b. \end{cases}$ Fig.9.

⊙ 11 $b^2 = (AD^2, u)\ AH^2.$

5 × 6 12 $\begin{cases} AI \times IK = (\text{ob Circl.})\ AL^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2. \end{cases}$

11 + 12 13 $\begin{cases} AD^2 + AL^2 = (DL^2, u)\ HL^2 = Q.\ Rad. \\ b^2 + \frac{S}{L^2} = Q.\ Rad. \end{cases}$

| | | | |
|---|---|---|---|
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 14 | $MO = -x.$ | |
| *Ob para.* | 15 | $\begin{cases} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{cases}$ | |
| *Ob para.* | 15 | $\begin{cases} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{cases}$ | |
| 15 − 10 | 16 | $\begin{cases} AO - AD = (DO, u)\, HO. \\ \frac{x^2}{L} (-b, u) - \frac{L}{2} + \frac{q}{2L} = (DO, u)\, HO. \end{cases}$ | *Fig.* 8. |
| 15 + 10 | 16 | $\begin{cases} AO + AD = (DO, u)\, HO. \\ \frac{x^2}{L} (+b, u) + \frac{q}{2L} - \frac{L}{2} = (DO, u)\, HO. \end{cases}$ | *Fig.* 9. |
| $\odot$ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = (DO^2, u)\, HO^2.$ | |
| *Q.* 14. | 18 | $x^2 = NO^2.$ | |
| *Q.* 14. | 18 | $x^2 = MO^2.$ | |
| 47, e 1<br>17 + 18 | 19 | $\begin{cases} DO^2 + NO^2 = HN^2. \\ b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} = HN^2 = Q.\ Rad. \end{cases}$ | |
| 47, e 1<br>17 + 18 | 19 | $\begin{cases} DO^2 + MO^2 = HM^2. \\ b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} = HM^2 = Q.\ Rad. \end{cases}$ | |
| 19 = 13 | 20 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 21 | $x^4 + qx^2 = S.$ | |
| *Transp.* | 22 | $x^4 + qx^2 - S = 0.$ *Q. e. d.* in Biquadratic. | *Fig.* 8, 9. |

*Illustrat.*

*q.* *s.*

$$\begin{cases} x^4 * + 60x^2 * - 11421 = 0 \\ x^4 * + 0.60x^2 * - 1.1421 = 0 \end{cases}$$

$$\begin{cases} NO = x = 9 \\ MO = -x = -9 \end{cases}$$

*Central.*

Central. C L A S.

$$\frac{L}{2} = 0.5$$

$$\frac{q}{2L} = 0.30$$

$$b = 0.20 = AD.$$

Fig. 8.

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{1.92}\,x^2 * - \overset{s.}{1.6384} = 0 \\ x^4 * + 1.92\,x^2 * - 1.6384 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 8 \\ MO = -x = -8 \end{array}\right\}$$

Central.

$$\frac{q}{2L} = 0.96$$

$$-\frac{L}{2} = 0.5$$

$$b = 0.46 = AD.$$

Fig. 9.

C L A S.

# CLAS. III.

*Of the Construction of Equations of three or four Dimension, where the second and third Term (viz. p and q) are wanting; or of Cubic Equations, affected under no Parodic Degree; or of Quadrato-quadratic, affected under the first Parodic Degree.*

ALL Equations of these kinds may be reduced to these following forms.

Fig. 10.

$$\left\{\begin{array}{l} 1.\ x^3 * * - r = o \left\{\begin{array}{l} 1\ x^4 * * - rx - S = o \\ 3\ x^4 * * - rx + S = o \end{array}\right. \\ 2.\ x^3 * * + r = o \left\{\begin{array}{l} 2\ x^4 * * + rx - S = o \\ 4\ x^4 * * + rx + S = o \end{array}\right. \end{array}\right\}$$

Fig. $\left\{\begin{array}{l}11\\12\end{array}\right.$

*Central Rule.*

*Synops. Cl.* 3.

$$\frac{L}{2} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Gen. Rule*

Suppose the Parabole (NAM) to be already described, whose *Latus Rectum* L (or 1), and Axe Ay;

2 1 in which let be taken $AD = \frac{L}{2}$; and erecting a Per-

4 2 pendicular to the Axe (*viz.* DH) $= \frac{r}{2L^2}$, from the Center H, must be described a Circle (NA), whose

6 3 Semidiameter HA, if it be only a Cubic Equation, (*i.e.*) if the Quantity S be not: But and if S be *Fig.* 10.

7 4 had, and it be $-$ S, then must be taken farther in this

8 5 Line AH, both ways produced, on the one side $AI = L$, *Fig.* 11.

9 6 and on the other side $AK = \frac{S}{L^3}$; and a Semicircle (ILK) being described, whose Diameter IK must

10 7 be erected AL perpendicular to AH, which may meet this Semicircle (ILK) in the Point L.

But

## CLAS. III.

*De Æquationibus trium vel quatuor Dimensionum construendis, ubi deficit secundus & tertius Terminus (nempè, p & q); vel, de Æquationibus Cubicis sub nullo; vel, de Quadrato-quadraticis, sub primo tantùm Gradu Parodico affectis.*

OMnes hujus censûs Æquationes ad sequentes formulas possint reduci.

Fig. 10. $\left\{\begin{array}{l} 1.\ x^3 * * - r = 0 \\ 2.\ x^3 * * + r = 0 \end{array}\right.$ $\left\{\begin{array}{ll} 1 & x^4 * * - rx - S = \\ 3 & x^4 * * - rx + S = \\ 2 & x^4 * * + rx - S = \\ 4 & x^4 * * + rx + S = \end{array}\right\}$ Fig. $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

*Regula Centralis.*

*Synops. Cl. 3.* $\frac{L}{2} = b = AD.\quad \frac{r}{2L^2} = d = DH.$

*Reg. Gen.* Supponatur Parabolam (NAM) jam descriptam esse, cujus Latus Rectum L (ceu 1), Axisque Ay;
2 1 in quo sumatur $AD = \frac{L}{2}$; & erigendo ad Axem per-
4 2 pendicularem $DH = \frac{r}{2L^2}$, oportet ex Centro H, Cir-
6 3 culum describere (NA), cujus Semidiameter sit HA, *Fig.* 10.
si Æquatio tantùm Cubica fuerit, hoc est, si non habea-
7 4 tur Quantitas S: Ast si habeatur S, & sit − S, opor-
tet ulteriùs in hâc lineâ AH, productâ utrinque, ex
8 5 unâ parte sumere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$, *Fig.* 11.
9 6 descriptoque Semicirculo (ILK), cujus Diameter IK,
10 7 erigere AL, perpendicularem ad AH, quæ occurrat huic Semicirculo (ILK) in puncto L.

Quòd

11 8 Quòd si verò habeatur $+ S$; oportet insuper in alio Semicirculo, cujus Diameter est AH, inscribere $AZ = AL$ inventæ. *Fig.* 12.

9 Circulus igitur descriptus transiens per L, si sit $-S$ (ut suprà § 7.); per Z verò si sit $+S$ (ut § 9.); secabit vel tanget Parabolam in 1 vel 2 punctis; à quibus si ad Axem demittantur Perpendiculares, obtinebuntur omnes Æquationis radices, tam falsæ, quàm veræ; nimirum, si in Æquatione habeatur $-r$; veræ, (ut NO,) ad sinistram partem Axis cadent, & falsæ (ut MO) ad dextram: Sed contra, si habeatur ibi $+r$, veræ cadent ad dextram Axis partem (ut MO), & falsæ (ut NO) ad sinistram.

(margin numbers beside this paragraph: 12, 13)

*Fig.* 10.

$$\left.\begin{array}{l} 1.\ x^3 * * - r = 0 \\ 2.\ x^3 * * + r = 0 \end{array}\right\} \quad \left\{\begin{array}{l} 1\ \}\ x^4 * * - rx - S = 0 \\ 3\ \}\ x^4 * * - rx + S = 0 \\ 2\ \}\ x^4 * * + rx - S = 0 \\ 4\ \}\ x^4 * * + rx + S = 0 \end{array}\right\}$$

*Fig.* 11, 12

---

*Demonstrat.*

1 10 $AD = \frac{L}{2} = b.$

2 11 $DH = \frac{r}{2L^2} = d.$

47, e 1 12 $AD^2 + DH^2 = (HA^2 =)$ Q. Rad.

Q. 10 + Q. 11. $b^2 + d^2 =$ Q. Rad. in Cubic. *Fig.* 10.

$AI \times AK =$ (ob Circle) $AL^2 =$ (per constr.) $AZ^2$.

5 × 6 13 $L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2.$

$AH^2 + AL^2 = (HL^2 =)$ Q. Rad.

47, e 1 12 + 13 14 $b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. in Biquadr. *Fig.* 11.

$AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad.

47, e 1 12 — 13 15 $b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. in Biquadr. *Fig:* 12.

NO

11

8 But if be had $+$ S; then moreover in another Semicircle, whose Diameter is A H, must be inscribed *Fig.* 12.
9 A Z = A L found. A Circle therefore described, passing through L, if it be — S (as above, § 7.); but through Z, if it be $+$ S (as § 9.) will cut or touch the Parabole in 1 or 2 Points; from which, if Perpendiculars be demitted to the Axe, all the Roots of the Equation, as well false as true, will be had; *viz.* The true (as N O) will fall to the left side of the Axe, and the false (as M O) to the right, if in the Equation be had $-$ r: But contrarily, if in it be had $+$ r, the true will fall to the right side of the Axe (as M O), and the false (as N O) to the left.

*Fig.* 10.

$$\left\{\begin{array}{l} 1.\ x^3 * * - r = 0 \\ 2.\ x^3 * * + r = 0 \end{array}\right. \quad \begin{array}{l} \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \left\{\begin{array}{l} x^4 * * - r x - S = 0 \\ x^4 * * - r x + S = 0 \end{array}\right\} \\ \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\} \left\{\begin{array}{l} x^4 * * + r x - S = 0 \\ x^4 * * + r x + S = 0 \end{array}\right\} \end{array}$$

*Fig.* $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

*Supp.* 16 $NO = x.$

*Supp.* 16 $MO = -x.$ *Fig.* 11.

*Ob para.* 17 $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$

*Ob para.* 17 $\left\{\begin{array}{l} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$

17 $\backsim$ 10 18 $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L}\ (\backsim b, u) \backsim \frac{L}{2} = HP. \end{array}\right.$

⊕ 19 $b^2 + \frac{x^4}{L^2} - x^2 = HP^2.$

16 $\backsim$ 11 20 $\left\{\begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x \backsim (d, u) \backsim \frac{r}{2L^2} = PN. \end{array}\right.$

| | | | |
|---|---|---|---|
| $16 + 11$ | 20 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| ☉ | 21 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ | |
| ☉ | 21 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, *e* 1.<br>$19 + 21$ | 22 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | |
| 47, *e* 1.<br>$19 + 21$ | 22 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | *Fig.* 11. |
| $22 = 12$ | 23 | $\frac{x^4}{L^2} - \frac{rx^2}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 24 | $x^3 * * - r = 0$. *Q. e. d.* in Cubic. | *Fig.* 10. |
| $22 = 14$ | 25 | $\frac{x^4}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 26 | $x^4 - rx = S.$ | |
| *Transp.* | 27 | $x^4 * * - rx - S = 0$. *Q. e. d.* in Biquadr. | *Fig.* 11. |
| $22 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - rx = -S.$ | |
| *Transp.* | 30 | $x^4 * * - rx + S = 0$. *Q. e. d.* in Biquadr. | *Fig.* 12. |
| *Supp.* | 16 | $MO = x.$ | *Fig.* 11. |
| *Supp.* | 16 | $NO = -x.$ | |
| *Ob para.* | 17 | $\left\{\begin{array}{l} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |
| *Ob para.* | 17 | $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |

| | | | |
|---|---|---|---|
| $10 \backsim 17$ | 18 | $AD \backsim AO = (DO, u)\ HP.$ | |
| | | $(b, u)\ \frac{L}{2} \backsim \frac{x^2}{L} = HP.$ | |
| $\odot$ | 19 | $b^2 + \frac{x^4}{L^2} - x^2 = HP^2.$ | |
| $16 + 11$ | 20 | $MO + (OP, u)\ DH = PM.$ | |
| | | $x\ (+d, u) + \frac{r}{2L^2} = PM.$ | |
| $16 \backsim 11$ | 20 | $NO \backsim (OP, u)\ DH = PN.$ | |
| | | $-x\ (-d, u) - \frac{r}{2L^2} = PN.$ | |
| $\odot$ | 21 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2 = PN^2.$ | |
| 47, *e* 1 | 22 | $HP^2 + PM^2 = (HM^2 =)$ Q. Rad. | |
| $19 + 21$ | | $b^2 + d^2 + \frac{x^4}{L^2} + \frac{rx}{L^2} =$ Q. Rad. | |
| $19 + 21$ | 22 | $HP^2 + PN^2 = (HN^2 =)$ Q. Rad. | |
| | | $b^2 + d^2 + \frac{x^4}{L^2} + \frac{rx}{L^2} =$ Q. Rad. | |
| $22 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 24 | $x^3 + rx = 0 = -NO.$ *Q.e.d.* in Cubic. | *Fig.* 10. |
| $22 = 14$ | 25 | $\frac{x^4}{L^2} + \frac{rx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 26 | $x^4 + rx = S.$ | |
| *Transp.* | 27 | $x^4 * * + rx - S = 0.$ *Q.e.d.* in Biquadratic. | *Fig.* 11. |
| $22 = 15$ | 28 | $\frac{x^4}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 + rx = - S.$ | |
| *Transp.* | 30 | $x^4 * * + rx + S = 0; -N = x.$ *Q.e.d.* in Biquadr. | *Fig.* 12. |

*Illustrat.*

$$\left.\begin{cases} x^3 ** - \overset{r.}{1728} = 0 \\ x^3 ** - 1.728 = 0 \end{cases}\right\} \quad NO = x = 12.$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.864 = d.$$

Fig. 10.

$$\left.\begin{cases} x^3 ** + \overset{r.}{1728} = 0 \\ x^3 ** + 1.728 = 0 \end{cases}\right\} \quad NO = -x = 12.$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.864 = d.$$

$$\left.\begin{cases} x^4 ** - \overset{r.}{1600}x - \overset{s.}{7761} = 0 \\ x^4 ** - 1.600x - 0.7761 = 0 \end{cases}\right\} \quad \begin{matrix} NO = x = 13. \\ MO = -x = -4.5 \end{matrix}$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.800 = d = DH.$$

Fig. 11.

$$\left.\begin{cases} x^4 ** + \overset{r.}{1600}x - \overset{s.}{7761} = 0 \\ x^4 ** + 1.600x - 0.7761 = 0 \end{cases}\right\} \quad \begin{matrix} MO = x = 4.5 \\ NO = -x = -13. \end{matrix}$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.800 = d = DH.$$

$$\begin{Bmatrix} x^4 * * - \overset{r.}{2560}x + \overset{s.}{9984} = 0 \\ x^4 * * - 2.560x + 0.9984 = 0 \end{Bmatrix} \quad \begin{matrix} NO = x = 12. \\ no = x = 4. \end{matrix}$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 1.280 = d = DH.$$

Fig. 12.

$$\begin{Bmatrix} x^4 * * + \overset{r.}{2560}x + \overset{s.}{9984} = 0 \\ x^4 * * + 2.560x + 0.9984 = 0 \end{Bmatrix} \quad \begin{matrix} NO = -x = -12. \\ no = -x = -4. \end{matrix}$$

*Central.*

$$\frac{L}{2} = 0.5 = b = AD. \quad \frac{r}{2L^2} = 1.280 = d = DH.$$

CLAS.

# CLAS. IV.

*De Æquationibus trium vel quatuor Dimensionum construendis, ubi deficit secundus Terminus (p); vel, de Æquationibus Cubicis, tantum sub primo; vel, de Quadrato-quadraticis, sub primo & secundo Gradu Parodico affectis.*

OMnes hujus Classis Æquationes ad hasce formulas sequentes reducuntur.

$$\left\{\begin{array}{l} 1.\ x^3 \ast - qx - r = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 \ast - qx^2 - rx - S = 0 \\ x^4 \ast - qx^2 - rx + S = 0 \end{array} \\ 2.\ x^3 \ast - qx + r = 0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\} \begin{array}{l} x^4 \ast - qx^2 + rx - S = 0 \\ x^4 \ast - qx^2 + rx + S = 0 \end{array} \end{array}\right\}$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 \ast + qx - r = 0 \left\{\begin{array}{l} 5 \\ 7 \end{array}\right\} \begin{array}{l} x^4 \ast + qx^2 - rx - S = 0 \\ x^4 \ast + qx^2 - rx + S = 0 \end{array} \\ 4.\ x^3 \ast + qx + r = 0 \left\{\begin{array}{l} 6 \\ 8 \end{array}\right\} \begin{array}{l} x^4 \ast + qx^2 + rx - S = 0 \\ x^4 \ast + qx^2 + rx + S = 0 \end{array} \end{array}\right\}$$

*Regula Centralis.*

*Synops. Cl. 4.* $$\frac{L}{2} \backsim \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Reg.Gen.* Supponendo itaque Parabolam (NAM) jam descriptam esse, cujus Latus Rectum esse L (ceu 1),
2 1 Axemque A y; in quo sumendo $A\,b = \frac{L}{2}$, oportet
3 2 facere $b\,D = \frac{q}{2L}$; eamque sumere in Axe continuato
3 deorsum versus y, si in Æquatione habeatur $-q$; sed versus alteram partem sursum, in eodem Axe continuato,
4 si habeatur $+q$.

Porrò

# CLAS. IV.

*Of the Construction of Equations of three or four Dimensions, where the second Term (viz. p) is wanting; or of Cubic Equations, affected only under the first Degree; or of Quadrato-quadratic, affected under the first and second Parodic Degree.*

ALL Equations of this Clas are reduced to these following forms.

$$\left\{\begin{array}{l} 1.\ x^3 * -qx - r = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * -qx^2 - rx - S = 0 \\ x^4 * -qx^2 - rx + S = 0 \end{array} \\ 2.\ x^3 * -qx + r = 0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\} \begin{array}{l} x^4 * -qx^2 + rx - S = 0 \\ x^4 * -qx^2 + rx + S = 0 \end{array} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 * +qx - r = 0 \left\{\begin{array}{l} 5 \\ 7 \end{array}\right\} \begin{array}{l} x^4 * +qx^2 - rx - S = 0 \\ x^4 * +qx^2 - rx + S = 0 \end{array} \\ 4.\ x^3 * +qx + r = 0 \left\{\begin{array}{l} 6 \\ 8 \end{array}\right\} \begin{array}{l} x^4 * +qx^2 + rx - S = 0 \\ x^4 * +qx^2 + rx + S = 0 \end{array} \end{array}\right.$$

*Central Rule.*

*Synops. Cl. 4.*

$$\frac{L}{2} \backsim \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Gen. Rule*

Supposing therefore a Parabole (NAM) already described, whose *Latus Rectum* is L (or 1), and Axe Ay;
2 1 in which taking $Ab = \frac{L}{2}$, must be made $bD = \frac{q}{2L}$, and
3 2 to place it on the Axe continued (downwards) towards y, if in the Equation be had $-q$; but (upwards) towards the other part, on the same Axe continued, if be had $+q$.
3 4

More-

Porrò è puncto D, erigendo ad Axem perpendicula-
3 rem $DH = \frac{r}{2L^2}$, oportet ex Centro H, Circulum
5 describere, cujus Semidiameter sit HA, si Æquatio tantum Cubica fuerit, hoc est, si non habeatur Quantitas S.

7 Ast si habeatur S, & sit quidem $-$ S, oportet ulterius in hac lineâ HA, utrinque productâ, ex
8 4 unâ parte sumere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$,
9 5 descriptoque Semicirculo, cujus Diameter IK, erigere
10 6 AL, perpendicularem ad HA, quæ occurrat huic Semicirculo (ILK) in puncto L.

11 Quòd si verò habeatur $+$ S, oportet insuper in alio Semicirculo, cujus Diameter sit AH, inscribere
7 $AZ = AL$ inventæ. Circulus igitur descriptus tran-
12 siens per L, si sit $-$ S (ut suprà § 6.); per Z verò si sit $+$ S (ut § 7.); secabit vel tanget Parabolam in tot punctis quot Æquatio admittet radices; à quibus si ad Axem demittantur perpendiculares, habebuntur omnes Æquationes radices, tàm falsæ, quàm veræ; quarum quidem veræ (ut NO) ad sinistram cadent, & falsæ
13 (ut MO) ad dextram partem Axis, si habeatur $-$ r: Sed contra, si habeatur ibi $+$ r; veræ cadent ad dextram (ut MO), & falsæ (ut NO) ad sinistram.

---

*Cas.* 1. Ubi $-$ q.

$$\left.\begin{array}{l}1.\\2.\end{array}\ x^3 * -qx \mp r = 0\right\}\begin{array}{l}1\ \ x^4 * -qx^2 \mp rx - S = 0\ \ 2\\3\ \ x^4 * -qx^2 \mp rx + S = 0\ \ 4\end{array}$$

*Demonstrat.*

$$\left\{\begin{array}{l}Ab + bD = b = AD.\\ \frac{L}{2} + \frac{q}{2L} = b = AD.\end{array}\right.$$ 8

9 $\frac{r}{2L^2} = d = DH.$

AD

Moreover from the Point D, erecting a Perpendi-
3 cular to the Axe $DH = \frac{r}{2L^2}$, must a Circle be de-
6 scribed from the Center H, whose Semidiameter HA,
if it be only a Cubic Equation, *viz.* if the Quantity S
be wanting.

7 But if S be had, and it be $-S$, there must farther in
this Line AH, both ways produced, be taken on the one
8 4 side $AI = L$, and on the other side $AK = \frac{S}{L^3}$; and a
9 5
10 6 Semicircle being described, whose Diameter IK,
must be erected AL perpendicular to HA, which may
meet this Semicircle (ILK), in the Point L.

11 But if $+S$ be had; then moreover in another
Semicircle, whose Diameter is AH, must be inscribed
7 $AZ = AL$ found. A Circle therefore described pas-
12 sing through L, if it be $-S$ (as above, § 6.); but
through Z, if it be $+S$, (as § 7.) will cut or touch
the Parabole, in as many Points, as the Equation will
admit Roots; from which, if Perpendiculars be de-
mitted to the Axe, all the Roots, as well false as
true, will be had; of which, the true (as NO) will
13 fall on the left side of the Axe, and the false (as MO)
on the right, if be had $-r$: But contrarily, if be had
$+r$; the true (as MO) will fall on the right, and
the false (as NO) on the left.

---

47, e 1 / Q. 8. + Q. 9. — 10 $\begin{cases} AD^2 + DH^2 = HA^2 = \text{Q. Rad.} \\ b^2 + d^2 = \text{Q. Rad. in Cubic.} \end{cases}$ Fig. 13.

$AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2.$

4 × 5 — 11 $L \times \frac{S}{L^3} = \frac{S}{L^2} = AL^2 = AZ^2.$

47, e 1 / 10 + 11 — 12 $\begin{cases} AH^2 + AL^2 = (HL^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{S}{L^2} = \text{Q. Rad. si} - S. \end{cases}$ Fig. 14.

*Fig.* 15.

| | | |
|---|---|---|
| 47, e 1<br>10 — 11 | 13 | $\left\{\begin{array}{l} AH^2 - AZ^2 = (HZ^2 =) \text{ Q. Rad.} \\ b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad. } h + S. \end{array}\right.$ |
| *Supp.* | 14 | $NO = x.$ |
| *Supp.* | 14 | $MO = - x.$ |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ |
| 15 — 8 | 16 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L}\ (\backsim b, u) - \frac{L}{2} - \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊖ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = HP^2.$ |
| 14 ∽ 9 | 18 | $\left\{\begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x \backsim (d, u) \backsim \frac{r}{2L^2} = PN. \end{array}\right.$ |
| 14 + 9 | 18 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ |
| ⊖ | 19 | $d^2 + x^2 - \frac{rx^2}{L^2} = PN^2.$ |
| ⊖ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ |
| 47, e 1.<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =) \text{ Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$ |
| 47, e 1.<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =) \text{ Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$ |

$\frac{x^4}{L^2}$

| | | | |
|---|---|---|---|
| $20 = 11$ | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * - qx - r = 0$. *Q. e. d.* in Cubic. | *Fig.* 13. |
| $20 = 12$ | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4 - qx^2 - rx = S$. | |
| *Transp.* | 25 | $x^4 * - qx^2 - rx - S = 0$. *Q. e. d.* si $- S$. | *Fig.* 14. |
| $20 = 13$ | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4 - qx^2 - rx = - S$. | |
| *Transp.* | 28 | $x^4 * - qx^2 - rx + S = 0$. *Q. e. d.* si $+ S$. | *Fig.* 15. |

$$2.\ x^3 * - qx + r = 0 \left\{ \begin{matrix} 2 \\ 4 \end{matrix} \right\} \left\{ \begin{matrix} x^4 * - qx^2 + rx - S = 0 \\ x^4 * - qx^2 + rx + S = 0 \end{matrix} \right\}$$

*Demonstrat.*

| | | |
|---|---|---|
| *Supp.* | 14 | $MO = x$. |
| *Supp.* | 14 | $NO = - x$. |
| *Ob para.* | 15 | $\left\{ \begin{matrix} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{matrix} \right.$ |
| *Ob para.* | 15 | $\left\{ \begin{matrix} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{matrix} \right.$ |
| $15 \backsim 8$ | 16 | $\left\{ \begin{matrix} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} (\backsim b, u) - \frac{L}{2} - \frac{q}{2L} = HP. \end{matrix} \right.$ |
| ☉ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = HP^2$. |
| $14 + 9$ | 18 | $\left\{ \begin{matrix} MO + (OP, u)\ DH = PM. \\ x (+ d, u) + \frac{r}{2L^2} = PM. \end{matrix} \right.$ |

14 $\backsim$ 9 | 18 | $\left\{\begin{array}{l} NO\ (\backsim OP,\ u)\ DH = PN. \\ -x\ (-d,\ u) - \frac{r}{2L^2} = PN. \end{array}\right.$

$\oplus$ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2.$

$\oplus$ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2.$

47, e 1 / 17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$

47, e 1 / 17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$

20 = 10 | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$.

$\times \frac{L^2}{x}$ | 22 | $x^3 * - qx + rx = 0$. *Q.e.d.* in Cubic. *Fig.* 13.

20 = 12 | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$.

$\times L^2$ | 24 | $x^4 - qx^2 + rx = S.$

*Transp.* | 25 | $x^4 * - qx^2 + rx - S = 0.$ *Q.e.d.* si $-S$. *Fig.* 14.

20 = 13 | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$.

$\times L^2$ | 27 | $x^4 - qx^2 + rx = -S.$

*Transp.* | 28 | $x^4 * - qx^2 + rx + S = 0$; *Q.e.d.* si $+S$. *Fig.* 15.

*Illustrat.*

$$\begin{array}{l} \quad\quad\quad\quad q. \quad\quad\quad r. \\ \left\{\begin{array}{l} x^3 * - 300x - 1703 = 0 \\ x^3 * - 3.00x - 1.703 = 0 \end{array}\right\} \end{array}$$

$$NO = x = 19.6 + \left\{\begin{array}{l} MO = -x = -13. \\ mo = -x = -\ 6.6 + \end{array}\right.$$

*Fig.* 13.

$$\begin{array}{l} \quad\quad\quad\quad q. \quad\quad\quad r. \\ \left\{\begin{array}{l} x^3 * - 300x + 1703 = 0 \\ x^3 * - 3.00x + 1.703 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} MO = x = 13. \\ mo = x = 6.6 \end{array}\right\} NO = -x = -19.6 +$$

Cen-

*Central.*

$$\left.\begin{aligned}&\frac{L}{2}=0.5\\&+\frac{q}{2L}=1.5\\&\overline{b=2.5}=AD\end{aligned}\right\}\ \frac{r}{2L^2}=0.8513=d=DH.$$

$$\begin{array}{cccc} & q. & r. & s.\end{array}$$

$$\left\{\begin{aligned}&x^4 * -144x^2-1200x-7600=0\\&x^4 * -1.44x^2-1.200x-0.7600=0\end{aligned}\right\}$$

$$NO = x = 15.8+$$

$$MO = -x = -10.$$

Fig. 14.

$$\begin{array}{cccc} & q. & r. & s.\end{array}$$

$$\left\{\begin{aligned}&x^4 * -144x^2+1200x-7600=0\\&x^4 * -1.44x^2+1.200x-0.7600=0\end{aligned}\right\}$$

$$MO = x = 10.$$

$$NO = -x = -15.8.$$

*Central.*

$$\left.\begin{aligned}&\frac{L}{2}=0.5\\&\frac{q}{2L}=0.72\\&\overline{b=1.22}=AD\end{aligned}\right\}\ \frac{r}{2L^2}=0.600=d=DH.$$

$$\begin{array}{cccc} & q. & r. & s.\end{array}$$

$$\left\{\begin{aligned}&x^4 * -376x^2-960x+18000=0\\&x^4 * -3.76x^2-0.960x+1.8000=0\end{aligned}\right\}$$

$$\left\{\begin{aligned}&NO=x=19.435+\\&no=x=6.\end{aligned}\right\}\ \left\{\begin{aligned}&MO=-x=-15.435\\&mo=-x=-10.\end{aligned}\right\}$$

Fig. 15.

$$\begin{array}{cccc} & q. & r. & s.\end{array}$$

$$\left\{\begin{aligned}&x^4 * -376x^2+960x+18000=0\\&x^4 * -3.76x^2+0.960x+1.8000=0\end{aligned}\right\}$$

$$\left\{\begin{aligned}&MO=x=15.435-\\&mo=x=10.\end{aligned}\right\}\ \left\{\begin{aligned}&NO=-x=-19.435\\&no=-x=-6.\end{aligned}\right\}$$

Cen-

*Central.*

$\frac{L}{2} + \frac{q}{2L} = 0.5 + 1.88 = 2.38 = b. \quad \frac{r}{2L^2} = 0.480 = d.$

*Cas.* 2. Ubi $+q$.

$$\begin{cases} 1. \ \frac{L}{2} \sqsupset \frac{q}{2L} \\ 2. \ \frac{q}{2L} \sqsupset \frac{L}{2} \end{cases}$$

3. $x^3 * + qx - r = 0 \begin{cases} 5 \\ 7 \end{cases} \begin{cases} \cancel{x^4 * + qx^2 - rx - S = 0} \\ x^4 * + qx^2 - rx + S = 0 \end{cases}$

*Demonstrat.*

| | | | |
|---|---|---|---|
| 1 — 2 | 8 | $\begin{cases} Ab - bD = AD = b. \\ \frac{L}{2} - \frac{q}{2L} = b = AD. \end{cases}$   1. Ubi $\frac{L}{2} \sqsupset \frac{q}{2L}$ | |
| 3 | 9 | $\frac{r}{2L^2} = d = DH.$ | |
| 47, e 1 $Q.$8. + $Q.$9. | 10 | $\begin{cases} AD^2 + DH^2 = (HA^2 =)\ Q.\ Rad. \\ b^2 + d^2 = Q.\ Rad.\ \text{in Cubic.} \end{cases}$ | *Fig.* 16. |
| 4 × 5 | 11 | $\begin{cases} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | |
| 47, e 1 10 + 11 | 12 | $\begin{cases} AH^2 + AL^2 = (HL^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{S}{L^2} = Q.\ Rad. \end{cases}$ In Biquadr. si $-S$. | *Fig.* 17. |
| 47, e 1 10 — 11 | 13 | $\begin{cases} AH^2 - AZ^2 = (HZ^2 =)\ Q.\ Rad. \\ b^2 + d^2 - \frac{S}{L^2} = Q.\ Rad. \end{cases}$ In Biquadr. si $+S$. | *Fig.* 18. |
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 15 | $MO = -x.$ | |

L . NO

| | | |
|---|---|---|
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ |
| $15 \backsim 8$ | 16 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L}\ (\backsim b, u) - \frac{L}{2} + \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊙ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = HP^2.$ |
| $14 \backsim 9$ | 18 | $\left\{\begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x\ (\backsim d, u) - \frac{r}{2L^2} = PN. \end{array}\right.$ |
| $14 + 9$ | 18 | $\left\{\begin{array}{l} MO + (PO, u)\ DH = PM. \\ -x\ (+ d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ |
| 47, e 1<br>$17 + 19$ | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ |
| 47, e 1<br>$17 + 19$ | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ |
| $20 = 10$ | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * + qx - r = 0$. *Q. e. d.* in Cubic. *Fig.* 16. |
| $20 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{s}{L^2}$; in $L^2$. |
| $\times L^2$ | 24 | $x^4 + qx^2 - rx = s.$ |
| *Transp.* | 25 | $x^4 * + qx^2 - rx \mp s = 0$. *Q. e. d.* in Biquadr. *Fig.* 17. |

$\frac{x^4}{L^2}$

| | | |
|---|---|---|
| 20 = 13 | 26 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| × $L^2$ | 27 | $x^4 + qx^2 - rx = -S$. |
| *Transp.* | 28 | $x^4 * + qx^2 - rx + S = 0$. *Q. e. d.* in Biquad. si $+ S$. *Fig.* 18. |
| | | 4. $x^3 * + qx + r = 0 \begin{cases} 6 \; x^4 * + qx^2 + rx - S = 0 \\ 8 \; x^4 * + qx^2 + rx + S = 0 \end{cases}$ |
| *Supp.* | 14 | $MO = x$. |
| *Supp.* | 14 | $NO = -x$. |
| *Ob para.* | 15 | $\begin{cases} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{cases}$ |
| *Ob para.* | 15 | $\begin{cases} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{cases}$ |
| 15 ∽ 8 | 16 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} (\backsim b, u) - \frac{L}{2} + \frac{q}{2L} = HP. \end{cases}$ |
| ⊚ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = HP^2$. |
| 14 + 9 | 18 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ x (+ d, u) + \frac{r}{2L^2} = PM. \end{cases}$ |
| 14 ∽ 9 | 18 | $\begin{cases} NO \backsim (OP, u)\ DH = PN. \\ -x (- d, u) - \frac{r}{2L^2} = PN. \end{cases}$ |
| ⊚ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2$. |
| ⊚ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2$. |
| 47, e. 1. 17 + 19 | 20 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.\ Rad. \end{cases}$ |

$\frac{x^4}{L^2}$ Pa

| | | | |
|---|---|---|---|
| 47, e 1 <br> $17+19$ | 20 | $\left\{ \begin{array}{l} HP^2+PN^2=(HN^2=)\ Q.\ Rad. \\ b^2+d^2+\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=Q.\ Rad. \end{array} \right.$ | |
| $20=10$ | 21 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 22 | $x^3 * +qx+rx=0$. *Q. e. d.* in Cubic. | *Fig.* 16. |
| $20=12$ | 23 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4+qx^2+rx=S$. | |
| *Transp.* | 25 | $x^4 * +qx^2+rx-S=0$. *Q. e. d.* in Biquadr. si $-S$. | *Fig.* 17. |
| $20=13$ | 26 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4+qx^2+rx=-S$. | |
| *Transp.* | 28 | $x^4 * +qx^2+rx+S=0$. *Q. e. d.* in Biquadr. si $+S$. | *Fig.* 18. |

*Illustrat.*

$$\left\{ \begin{array}{l} x^3 * + \overset{q.}{50}x - \overset{r.}{912}=0 \\ x^3 * + 0.50x - 0.912=0 \end{array} \right\} \quad NO=x=8.$$

*Fig.* 16.

$$\left\{ \begin{array}{l} x^3 * + \overset{q.}{50}x + \overset{r.}{912}=0 \\ x^3 * + 0.50x + 0.912=0 \end{array} \right\} \quad NO=-x=-8.$$

*Central.*

$$\left. \begin{array}{r} \frac{L}{2}=0.5 \\ -\frac{q}{2L}=0.25 \\ \hline b=0.25=AD \end{array} \right\} \quad \frac{r}{2L^2}=0.456=d=DH.$$

$$\left\{ \begin{array}{l} x^4 * + \overset{q.}{50}x^2 - \overset{r.}{1200}x - \overset{s.}{13536}=0 \\ x^4 * + 0.50x^2 - 1.200x - 1.3536=0 \end{array} \right\}$$

*Fig.* 17.

$NO=x=12.$

$MO=-x=-7.1-$

H x⁴ *

$$\begin{Bmatrix} & & q. & & r. & & s. \\ x^4 & * & +050x^2 & + & 1205x & - & 13536=0 \\ x^4 & * & +0.50x^2 & + & 1.200x & - & 1.3536=0 \end{Bmatrix}$$ *Fig.* 17.

MO $= x =$ 7.1 *ferè.*

NO $= -x = -12.$

*Central.*

$$\left.\begin{array}{l} \frac{L}{2}=0.5 \\ -\frac{q}{2L}=0.25 \\ \hline b=0.25=AD \\ \hline \end{array}\right\} \frac{r}{2L^2}=0.6co=d=DH.$$

$$\begin{Bmatrix} & & q. & & r. & & s. \\ x^4 & * & +50x^2 & - & 2828x & + & 6000=0 \\ x^4 & * & +0.50x^2 & - & 2.828x & + & 0.6000=0 \end{Bmatrix}$$

NO $= x = 12.$

no $= x =$ 2.2 $+$

$$\begin{Bmatrix} & & q. & & r. & & s. \\ x^4 & * & +50x^2 & + & 2828x & + & 6000=0 \\ x^4 & * & +0.50x^2 & + & 2.828x & + & 0.6c00=0 \end{Bmatrix}$$ *Fig.* 18.

NO $= -x = -12.$

no $= -x = -2.2$

*Central.*

$$\left.\begin{array}{l} \frac{L}{2}=0.5 \\ -\frac{q}{2L}=0.25 \\ \hline b=0.25=b=AD. \\ \hline \end{array}\right\} \frac{r}{2L^2}=1.414=d=DH.$$

2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2}$.

3. $x^3 * + qx - r = 0 \begin{Bmatrix} 5 \\ 7 \end{Bmatrix} \begin{Bmatrix} x^4 * + qx^2 - rx - S = 0 \\ x^4 * + qx^2 - rx + S = 0 \end{Bmatrix}$

*Demonstr.*

*Demonstrat.*

| | | | |
|---|---|---|---|
| 2 — 1 | 8 | $bD - Ab = b = AD.$<br>$\frac{q}{2L} - \frac{L}{2} = b = AD.$ | |
| 3 | 9 | $\frac{r}{2L^2} = d = DH.$ | |
| 47, e 1<br>*Q.* 8. +<br>*Q.* 9. | 10 | $AD^2 + DH^2 = (HA^2)$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic, | *Fig.* 19. |
| 4 × 5 | 11 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2$.<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | |
| 47, e 1<br>10 + 11 | 12 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad.<br>$b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad.   } In Biquadr. si — S. | *Fig.* 20. |
| 47, e 1<br>10 — 11 | 13 | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad.<br>$b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad.   } In Biquadr. si + S. | *Fig.* 21. |
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 14 | $MO = -x.$ | |
| *Ob para.* | 15 | $L \,.\, NO :: NO \,.\, AO.$<br>$L \,.\, x :: x \,.\, \frac{x^2}{L} = AO.$ | |
| *Ob para.* | 15 | $L \,.\, MO :: MO \,.\, AO.$<br>$L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO.$ | |
| 15 + 8 | 16 | $AO + AD = (DO, u)\ HP.$<br>$\frac{x^2}{L}\ (+ b, u) + \frac{q}{2L} - \frac{L}{2} = HP.$ | |
| ☉ | 17 | $b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - x^2 = HP^2.$ | |
| 9 ∽ 14 | 18 | $(OP, u)\ DH \backsim NO = PN.$<br>$(d, u)\ \frac{r}{2L^2} - x = PN.$ | |

| | | | |
|---|---|---|---|
| 14 + 9 | 18 | $\begin{cases} MO + (PO,u)\, DH = PM. \\ -x\ (+d,u) + \frac{r}{2L^2} = PM. \end{cases}$ | |
| ⊖ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ | |
| ⊖ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.\,Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.\,Rad. \end{cases}$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.\,Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.\,Rad. \end{cases}$ | |
| 20 = 10 | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * + qx - r = 0.$ *Q. e. d.* in Cubic. | *Fig.* 19. |
| 20 = 12 | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4 + qx^2 - rx = S.$ | |
| *Transp.* | 25 | $x^4 * + qx^2 - rx - S = 0.$ *Q. e. d.* in Biquadr. si $-S$. | *Fig.* 20. |
| 20 = 13 | 26 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4 + qx^2 - rx = -S.$ | |
| *Transp.* | 28 | $x^4 * + qx^2 - rx + S = 0.$ *Q. e. d.* in Biquad. si $+S$. | *Fig.* 21. |
| | | 4. $x^3 * + qx + r = 0 \begin{cases} 6 & x^4 * + qx^2 + rx - S = 0 \\ 8 & x^4 * + qx^2 + rx + S = 0 \end{cases}$ | |
| *Supp.* | 14 | $MO = x.$ | |
| *Supp.* | 14 | $NO = -x.$ | |
| *Ob para.* | 15. | $\begin{cases} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{cases}$ | |

L . NO

| | | | |
|---|---|---|---|
| *Ob para* | 15 | $\begin{cases} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{cases}$ | |
| $15 \backsim 8$ | 16 | $\begin{cases} AO + AD = (DO, u)\ HP. \\ \frac{x^2}{L}\ (+b, u) + \frac{q}{2L} - \frac{L}{2} = HP. \end{cases}$ | |
| $\ominus$ | 17 | $b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - x^2 = HP^2.$ | |
| $14 + 9$ | 18 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ x\ (+d, u) + \frac{r}{2L^2} = PM. \end{cases}$ | |
| $9 \backsim 14$ | 18 | $\begin{cases} (OP, u)\ DH \backsim NO = PN. \\ (d, u)\ \frac{r}{2L^2} + x = PN. \end{cases}$ | |
| $\ominus$ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2.$ | |
| $\ominus$ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2.$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{cases}$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{cases}$ | |
| $20 = 10$ | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * + qx + r = 0$. *Q. e. d.* in Cubic. | *Fig.* 19. |
| $20 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4 + qx^2 + rx = S.$ | |
| *Transp.* | 25 | $x^4 * + qx^2 + rx - S = 0$. *Q. e. d.* in Biquadr. si $-S$. | *Fig.* 20. |
| $20 = 13$ | 26 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4 + qx^2 + rx = -S.$ | |
| *Transp.* | 28 | $x^4 * + qx^2 + rx + S = 0$. *Q. e. d.* in Biquadr. si $+S$. | *Fig.* 21. |

*Illustrat.*

*Illustrat.*

$$\left\{\begin{array}{l} \overset{}{x^3 * +} \overset{q.}{384}x - \overset{r.}{3584} = 0 \\ x^3 * + 3.84x - 3.584 = 0 \end{array}\right\} \quad NO = x = 8.$$

Fig. 19.

$$\left\{\begin{array}{l} x^3 * + \overset{q.}{384}x + \overset{r.}{3584} = 0 \\ x^3 * + 3.84x + 3.584 = 0 \end{array}\right\} \quad NO = -x = -8.$$

*Central.*

$$\left.\begin{array}{l} \frac{q}{2L} = 1.92 \\ -\frac{L}{2} = 0.5 \\ \hline b = 1.42 = AD. \\ \hline \end{array}\right\} \frac{r}{2L^2} = 1.792 = d = DH.$$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{396}x^2 - \overset{r.}{3256}x - \overset{s.}{17040} = 0 \\ x^4 * + 3.96x^2 - 3.256x - 1.7040 = 0 \end{array}\right\}$$

$NO = x = 10.$

$MO = -x = -3.6$ &c.

Fig. 20.

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{396}x^2 + \overset{r.}{3256}x - \overset{s.}{17040} = 0 \\ x^4 * + 3.96x^2 + 3.256x - 1.7040 = 0 \end{array}\right\}$$

$MO = x = 3.6$ &c.

$NO = -x = -10.$

Hactenus de Æquationibus quartum gradum non excedentibus, ſub nullo extremo gradu Parodico affectis; vel, ubi deficit quantitas (p); in quibus omnes Rectæ ad Axem applicantur: Jam de reliquis, ubi omnes rectæ ad Diametrum ſunt applicandæ.

CLAS.

*Central.*

$$\left.\begin{array}{l}\frac{q}{2L} = 1.98\\ \frac{L}{2} = 0.5\\ \hline b = 1.48 = AD.\end{array}\right\}\ \frac{r}{2L^2} = 1.628 = d = DH.$$

$$\left\{\begin{array}{l} \overset{}{x^4} * + \overset{q.}{200\,x^2} - \overset{r.}{3600\,x} + \overset{s.}{6000} = 0\\ x^4 * + 2.00\,x^2 - 3.600\,x + 0.6000 = 0\end{array}\right\}$$

$NO = x = 10.$

$no = x = 1.8 +$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{200\,x^2} + \overset{r.}{3600\,x} + \overset{s.}{6000} = 0\\ x^4 * + 2.00\,x^2 + 3.600\,x + 0.6000 = 0\end{array}\right\}$$

$NO = -x = -10.$

$no = -x = -1.8 +$

*Central.*

$$\left.\begin{array}{l}\frac{q}{2L} = 1.00\\ \frac{L}{2} = 0.5\\ \hline b = 0.5 = AD.\end{array}\right\}\ \frac{r}{2L^2} = 1.800 = d = DH.$$

Fig.20.

Hitherto have we treated of Æquations, not exceeding the fourth degree, affected under no extream Parodic; or, where the Quantity (p) is wanting; in which all Right Lines are referred to the Axe: Now, of the rest, where p is had, where all Right Lines are to be applied to the Diameter.

C L A S.

# CLAS. V.

*De Æquationibus linearibus; & Quadrato-quadraticis construendis, & Radicibus Geometricè investigandis, affectis tantum sub tertio Gradu Parodico; vel, de Æquationibus unius, vel quatuor Dimensionum, ubi deficiunt tertius & quartus Terminus (q & r.)*

AD hasce sequentes formulas omnes hujus generis Æquationes reduci possint.

$$\left\{\begin{array}{l} 1.\ x - p = 0 \left\{\begin{array}{l} 1 \}\ x^4 - p x^3 * * - S = 0 \\ 3 \}\ x^4 - p x^3 * * + S = 0 \end{array}\right. \\ 2.\ x + p = 0 \left\{\begin{array}{l} 2 \}\ x^4 + p x^3 * * - S = 0 \\ 4 \}\ x^4 + p x^3 * * + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} = d = DH.$$

*Reg. Gen.* Supponatur itaque Parabolam (NAM) jam descriptam esse, cujus Latus Rectum sit L (seu 1), Axisque (A y); ad quem ordinatim applicetur Recta

1 1 $BA = \frac{p}{2}$, occurrens Parabolæ in B & A: Et ex A (puta) ducatur Diameter, vel Axi parallela A y; in

2 2 quâ sumendo $Ab = \frac{L}{2}$, oportet facere $bD = \frac{p^2}{8L}$,

2 3 eamque sumere in Diametro continuatâ deorsum versus y; (vel sic, sumatur $AD = \frac{L}{2} + \frac{p^2}{8L}$, eamque deorsum in Diametro, collocando.

Porrò

## CLAS. V.

*Of the construction of Linear Equations, and of Quadrato-quadratics, and of the invention of their Roots geometrically, affected only under the third Parodic Degree; or of Equations, of one only, or of four Dimensions, where the third and fourth Terms are wanting,* (viz. *q and r.*)

ALL Equations of these sorts may be reduced to these following forms.

$$\left\{\begin{array}{l} 1.\ x - p = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\}\begin{array}{l} x^4 - p x^3 * * - S = 0 \\ x^4 - p x^3 * * + S = 0 \end{array} \\ 2.\ x + p = 0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\}\begin{array}{l} x^4 + p x^3 * * - S = 0 \\ x^4 + p x^3 * * + S = 0 \end{array} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} = d = DH.$$

Gen. Rule | Let the Parabole (NAM) therefore be supposed to be already described, whose *Latus Rectum* is L (or 1), and Axe (A y); to which, let the Ordinate

1 | 1 | $BA = \frac{p}{2}$ be applied, meeting the Parabole in B and A: And from A (suppose) draw a Diameter, or a Parallel to the Axe (*viz.* A y); in which, making

2 | 2 | $Ab = \frac{L}{2}$, must be made $bD = \frac{p^2}{8L}$, setting it down-

2 | 3 | wards on the Diameter continued towards y; (or you may make $AD = \frac{L}{2} + \frac{p^2}{8L}$, and place it downwards on the Diameter.

I

Then

Porrò ex D, erigenda est ad Diametrum perpendi-
4 4 cularis $DH = \frac{p}{4} + \frac{p^3}{16L^2}$; (vel sumatur $De = \frac{p}{4}$, &
5 ulterius ad sinistram, $eH = \frac{p^3}{16L^2}$;) Tum ex Centro
H, oportet Circulum describere, cujus Semidiame-
6 6 ter sit H A, si in Æquatione non habeatur Quanti- *Fig.*22.
tas S.

7 Ast si habeatur S, & sit quidem $-$ S, oportet ulte-
riùs in hâc lineâ H A, utrinque productâ, ex unâ
8 7 8 9 parte sumere $AI = L$; & ex alterâ parte, $AK = \frac{S}{L^3}$;
descriptoque Semicirculo, cujus Diameter I K, eri-
10 9 gere A L perpendicularem ad A H, quæ occurrat huic
Semicirculo (I L K) in puncto L.

11 Quòd si verò habeatur $+$ S; oportet insuper in
alio Semicirculo, cujus Diameter sit H A, inscribere
10 $AZ = AL$, inventæ. Circulus igitur descriptus, tran- *Fig.*23.
12 siens per L, si sit $-$ S; per Z verò, si sit $+$ S, secabit *Fig.*24.
vel tanget Parabolam, in 1 ceu 2 punctis, à quibus si ad
Diametrum demittantur Perpendiculares, obtinebun-
tur omnes Æquationes radices, tam falsæ, quam veræ.
Quarum quidem veræ (ut N O) ad sinistram cadent,
14 & falsæ (ut M O) ad dextram, si in Æquatione ha-
15 beatur $-$ p: Sed contra, si habeatur ibi $+$ p, veræ qui-
dem cadent (ut M O) ad dextram, falsæ verò (ut N O)
ad sinistram.

---

$$1.\ x - p \pm 0 \left\{ \begin{matrix} 1 \\ 3 \end{matrix} \right\} \begin{matrix} x^4 - px^3 * * - S = 0 \\ x^4 - px^3 * * + S = 0 \end{matrix}$$

*Demonstrat.*

2 + 3 11
$$\left\{ \begin{matrix} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} = b = AD. \end{matrix} \right.$$

4 + 5 12
$$\left\{ \begin{matrix} De + eH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} = d = DH. \end{matrix} \right.$$

Then from D, erect a Perpendicular to the Diameter
4 4 $DH = \frac{p}{4} + \frac{p^3}{16L^2}$; (or take $De = \frac{p}{4}$, and farther
5 towards the left hand make $eH = \frac{p^3}{16L^2}$:) Then from
the Center H must a Circle be described, whose Semi-
6 6 diameter is HA, if in the Equation the Quantity S be not had. *Fig.* 22

7 But if S be had, and it be $-S$, then must there far-
ther in this Line AH, both ways produced, be taken on
8 7 8 the one side $AI = L$, and on the other side $AK = \frac{S}{L^3}$;
9 and a Semicircle being described, whose Diameter IK,
10 9 must be erected AL perpendicular to AH, which may
meet this Semicircle (ILK) in the Point L.

11 But if $+S$ be had; then moreover in another
Semicircle, whose Diameter is HA, must be inscribed
10 $AZ = AL$ found. A Circle therefore described paf- *Fig.* 23.
12 sing through L, if it be $-S$; but through Z, if it be *Fig.* 24.
$+S$, will cut or touch the Parabole in 1 or 2 Points;
from which, if Perpendiculars be demitted to the Diameter, all the Roots of the Equation, as well false as
true, will be had; of which, the true (as NO) will
14 fall to the left hand, and the false (as MO) on the
right, if in the Equation be had $-p$: But if in it be
had $+p$, then the true Roots (as MO) will fall on
the right hand, but the false (as NO) to the left.

---

47, *e* 1
*Q*. 11. + 13 $\left\{ \begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = HA^2. \end{array} \right.$
*Q*. 12.

7 × 8 14 $\left\{ \begin{array}{l} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array} \right.$ *Fig.* 22.

47, *e* 1
13 + 14 15 $\left\{ \begin{array}{l} HA^2 + AL^2 = (HL^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{S}{L^2} = Q.\ Rad. \end{array} \right\}$ In Biquadr. fi $-S$. *Fig.* 23.

| | | | |
|---|---|---|---|
| 47, e 1<br>13 − 14 | 16 | $\left\{\begin{array}{l} HA^2 - AZ^2 = (HZ^2 =)\ Q.\ Rad. \\ b^2 + d^2 - \frac{S}{L^2} = Q.\ Rad. \end{array}\right\}$ In Biquadr. si + S. | *Fig.* 24. |
| *Supp.* | 17 | $NO = x.$ | |
| *Supp.* | 17 | $MO = -x.$ | |
| 17 − 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (FN, u)\ OR \\ x - \frac{p}{2} = OR. \end{array}\right.$ | |
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (FM, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | |
| 19 ∽ 11 | 20 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{px}{8L} = HP. \end{array}\right.$ | |
| ☉ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^2x}{8L^2} = HP^2.$ | |
| 17 ∽ 12 | 22 | $\left\{\begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^2}{16L^2} = PN. \end{array}\right.$ | |
| 17 + 12 | 22 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^2}{16L^2} = PM. \end{array}\right.$ | |
| ☉ | 23 | $d^2 + x^2 - \frac{px}{2} - \frac{p^2x}{8L^2} = PN^2.$ | |
| ☉ | 23 | $d^2 + x^2 - \frac{px}{2} - \frac{p^2x}{8L^2} = PN^2.$ | |
| 21 + 23 | 24 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} = Q.\ Rad. \end{array}\right.$ | $\frac{x^4}{L^2}$ |

| | | | |
|---|---|---|---|
| 24 = 13 | 25 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} = 0$; in $\frac{L^2}{x^3}$. | |
| $\times \frac{L^2}{x^3}$ | 26 | $x - p = 0$. *Q. e. d.* in Linear. | *Fig.* 22. |
| 24 = 15 | 27 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 28 | $x^4 - px^3 = S$. | |
| *Transp.* | 29 | $x^4 - px^3 * * - S = 0$. *Q. e. d.* in Biquadr. si $- S$. | *Fig.* 23. |
| 24 = 16 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 = - S$. | |
| *Transp.* | 32 | $x^4 - px^3 * * + S = 0$. *Q. e. d.* in Biquad. si $+ S$. | *Fig.* 24. |
| | | 2. $x + p = 0 \begin{cases} 3 \; x^4 + px^3 * * - S = 0 \\ 4 \; x^4 + px^3 * * + S = 0 \end{cases}$ | |
| *Supp.* | 17 | $MO = x$ | |
| *Supp.* | 17 | $NO = - x$ | |
| 17 + 1 | 18 | $\begin{cases} MO + (OF, u)\, BA = (FM, u)\, OR \\ x + \frac{p}{2} = OR. \end{cases}$ | |
| 17 − 1 | 18 | $\begin{cases} NO - (OF, u)\, BA = (FN, u)\, OR. \\ - x - \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| 19 $\backsim$ 11 | 20 | $\begin{cases} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\, (-b, u) - \frac{L}{2} - \frac{p^2}{8L} = HP. \end{cases}$ | |
| ⊕ | 21 | $\begin{cases} b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2. \\ \text{he, } b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP. \end{cases}$ | |

| | | | |
|---|---|---|---|
| $17+12$ | 22 | $\begin{cases} MO + (OP, u)\, DH = PM. \\ x\,(+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} = PM. \end{cases}$ | |
| $17-12$ | 22 | $\begin{cases} NO - (OP, u)\, DH = PN. \\ -x\,(-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} = PN. \end{cases}$ | |
| ⊙ | 23 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3 x}{8L^2} = PM^2.$ | |
| ⊙ | 23 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3 x}{8L^2} = PN^2.$ | |
| 47, e 11<br>$21+23$ | 24 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} = Q.\ Rad.\ (= HN^2.) \end{cases}$ | |
| $24 = 13$ | 25 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = 0$; in $\frac{L^2}{x^3}$. | |
| $\times \frac{L^2}{x^3}$ | 26 | $x + p = 0$. *Q. e. d.* in Linear. | *Fig.* 22. |
| $24 = 15$ | 27 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 28 | $x^4 + px^3 = S.$ | |
| *Transp.* | 29 | $x^4 + px^3 * * - S = 0$. *Q. e. d.* in Biquadr. si $-S$. | *Fig.* 23. |
| $24 = 16$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 = -S.$ | |
| *Transp.* | 32 | $x^4 + px^3 * * + S = 0$. *Q. e. d.* in Biquadr. si $+S$. | *Fig.* 24. |

*Illustrat.*

$$\begin{cases} 1.\ x - \overset{p}{1.6} = 0 \\ 2.\ x + 1.6 = 0 \end{cases} \quad \begin{matrix} NO = x = 16. \\ NO = -x = -16. \end{matrix}$$

$$\begin{cases} \frac{p}{2} = 0{,}8. \quad \frac{p^2}{4} = 0.64. \quad \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. \quad \frac{p^2}{8} = 0.32. \quad \frac{p^3}{16} = 0.256. \end{cases}$$ *Fig.* 22.

Central.

*Central.* *Fig.* 22.

$$\left.\begin{array}{l} \dfrac{L}{2} = 0.5 \\ \dfrac{p^2}{8L} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{l} \dfrac{p}{4} = 0.4 \\ \dfrac{p^3}{16L^2} = 0.256 \\ \hline d = 0.656 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{10}x^3 * * - \overset{s.}{3456} = 0 \\ x^4 - 1.0x^3 * * - 0.3456 = 0 \end{array}\right\} \quad \begin{array}{l} NO = x = 12. \\ MO = -x = -6. \end{array}$$

$$\left\{\begin{array}{l} x^4 + \overset{p.}{10}x^3 * * - \overset{s.}{3456} = 0 \\ x^4 + 1.0x^3 * * - 0.3456 = 0 \end{array}\right\} \quad \begin{array}{l} MO = x = 6. \\ NO = -x = -12. \end{array}$$

*Fig.* 23.

$$\left\{\begin{array}{lll} \dfrac{p}{2} = 0.5. & \dfrac{p^2}{4} = 0.25. & \dfrac{p^3}{8} = 0.125 \\ \dfrac{p}{4} = 0.25. & \dfrac{p^2}{8} = 0.125. & \dfrac{p^3}{16} = 0.0625. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{l} \dfrac{L}{2} = 0.5 \\ \dfrac{p^2}{8L} = 0.125 \\ \hline b = 0.625 = AD \end{array}\right\} \quad \begin{array}{l} \dfrac{p}{4} = 0.25 \\ \dfrac{p^3}{16L^2} = 0.0625 \\ \hline d = 0.3125 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{15}x^3 * * + \overset{s.}{2744} = 0 \\ x^4 - 1.5x^3 * * + 0.2744 = 0 \end{array}\right\} \quad \begin{array}{l} NO = x = 14. \\ no = x = 7. \end{array}$$

$$\left\{\begin{array}{l} x^4 + \overset{p.}{15}x^3 * * + \overset{s.}{2744} = 0 \\ x^4 + 1.5x^3 * * + 0.2744 = 0 \end{array}\right\} \quad \begin{array}{l} NO = -x = -14. \\ no = -x = -7. \end{array}$$ *Fig.* 24.

*Central.*

$$\frac{L}{2} + \frac{p^2}{8L} = 0.78125 = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} = 0.5859275 = d = DH$$

CLAS.

# CLAS. VI.

*De Æquationibus Cubicis, affectis tantùm sub Quadrato, vel secundo gradu Parodico; vel de Quadrato-quadraticis, affectis tantùm sub Latere & Cubo; vel sub primo & tertio gradu Parodico; seu de Æquationibus tertiæ vel quartæ Dimensionis, ubi deficit tertius Terminus, vel Quantitas (q).*

HUjus censûs Æquationes ad sequentes formulas reducuntur; sc.

$$\left\{\begin{array}{l} 1.\ x^3 - px^2 * + r = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right. \begin{array}{l} x^4 - px^3 * + rx - S = 0 \\ x^4 - px^3 * + rx + S = 0 \end{array} \\ 2.\ x^3 + px^2 * - r = 0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right. \begin{array}{l} x^4 + px^3 * - rx - S = 0 \\ x^4 + px^3 * - rx + S = 0 \end{array} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 * - r = 0 \left\{\begin{array}{l} 5 \\ 7 \end{array}\right. \begin{array}{l} x^4 - px^3 * - rx - S = 0 \\ x^4 - px^3 * - rx + S = 0 \end{array} \\ 4.\ x^3 + px^2 * + r = 0 \left\{\begin{array}{l} 6 \\ 8 \end{array}\right. \begin{array}{l} x^4 + px^3 * + rx - S = 0 \\ x^4 + px^3 * + rx + S = 0 \end{array} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH.$$

*Reg. Gen.* Describatur itaque Parabola (NAM), cujus Latus Rectum sit L (seu 1), & Axis (ay); ad quem or-
1 dinatìm applicetur Recta BA $= \frac{p}{2}$, occurrens Parabolæ in B & A: Ex A (puta) ducatur Diameter, vel
2 2 Recta Axi parallela (Ay); in quâ sumendo Ab $= \frac{L}{2}$, oportet

# CLAS. VI.

*Of Cubic Equations, affected only under a Square, or the second Parodic Degree; or of Quadrato-quadratics, affected only under a Side and a Cube, or under the first and third Parodic Degree; or of Equations of the third or fourth Dimension, where the third Term or Quantity (q) is wanting.*

ALL Equations of this kind are reduced to these following forms, *viz.*

$$\left\{\begin{array}{ll} 1.\ x^3 - px^2 * + r = 0 & \left\{\begin{array}{l} 1\ \}\ x^4 - px^3 * + rx - S = 0 \\ 3\ \}\ x^4 - px^3 * + rx + S = 0 \end{array}\right\} \\ 2.\ x^3 + px^2 * - r = 0 & \left\{\begin{array}{l} 2\ \}\ x^4 + px^3 * - rx - S = 0 \\ 4\ \}\ x^4 + px^3 * - rx + S = 0 \end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{ll} 3.\ x^3 - px^2 * - r = 0 & \left\{\begin{array}{l} 5\ \}\ x^4 - px^3 * - rx - S = 0 \\ 7\ \}\ x^4 - px^3 * - rx + S = 0 \end{array}\right\} \\ 4.\ x^3 + px^2 * + r = 0 & \left\{\begin{array}{l} 6\ \}\ x^4 + px^3 * + rx - S = 0 \\ 8\ \}\ x^4 + px^3 * + rx + S = 0 \end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH.$$

*Gen. Rule* Let a Parabole (NAM) therefore be described, whose *Latus Rectum* L (or 1), and Axe (a y); to
1 1 which, let be ordinately applied $BA = \frac{p}{2}$, meeting the Parabole in B and A: From A (suppose) let there be drawn a Diameter, or a Right Line, parallel to
2 2 the Axe (*viz.* A y); in which, taking $Ab = \frac{L}{2}$,

2 3 oportet facere $bD = \frac{p^2}{8L}$ (vel $AD = \frac{L}{2} + \frac{p^2}{8L}$) eam-
que collocare in Diametro continuatâ versùs y.

Tum è Puncto D, erigatur ad Diametrum perpendicularis $DH = \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$, si p & r iisdem signis sint affectæ, quæ ad sinistram est collocanda; sin p & r diversis Signis fuerint affectæ, & $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}$, tum fiat $DH = \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2}$, quæ etiam ad sinistram est collocanda: Quòd si $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$; tum fiat $DH = \frac{r}{2L^2} - \frac{p}{4} - \frac{p^3}{16L^2}$, quæ ad dextram est collocanda.

Vel sic: Erigendo ad Diametrum perpendicularem
4 4 (DH), oportet in eâ sumere $De = \frac{p}{4}$, & ulteriùs
5 in eâ versus sinistram, oportet facere $ef = \frac{p^3}{16L^2}$; imò,
si in Æquatione p & r, iisdem signis sint adfectæ, ulte-
6 6 riùs ad sinistram oportet facere $fH = \frac{r}{2L^2}$; ad dex-
tram verò (ex Puncto f), si p & r, diversis Signis
sint denotatæ.

Tum Centro H, intervallo verò HA, describatur Circulus (NAM), si Æquatio fuerit tantùm Cubica, hoc est, si non habeatur Quantitas S. *Fig.* 25.

Ast si habeatur S, & sit quidem $-S$, oportet ulte-
riùs in hâc lineâ HA, utrinque productâ, ex unâ
8 7 9 8 parte sumere $AI = L$; & ex alterâ parte, $AK = \frac{S}{L^3}$;
describtoque Semicirculo, cujus Diameter IK, eri-
10 9 gere AL perpendicularem ad AH, quæ occurrat huic
Semicirculo (ILK) in puncto L.
11 Quòd si verò habeatur $+S$; oportet insuper in
alio Semicirculo, cujus Diameter sit HA, inscribere
11 10 $AZ = AL$, inventæ. Circulus igitur descriptus, tran-
12 siens per L, si sit $-S$; per Z verò, si sit $+S$, secabit
vel

2 3 must be made $bD = \frac{p^2}{8L}$, (or $AD = \frac{L}{2} + \frac{p^2}{8L}$) placing it in the Diameter continued towards y.

Then from the Point D, erect perpendicularly to the Diameter $DH = \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$, if p and r are affected with the same Signs, which is to be placed to the left hand; but if p and r are affected with divers Signs, and $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}$, then make $DH = \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2}$, and place it also to the left: But if $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$, then make $DH = \frac{r}{2L^2} - \frac{p}{4} - \frac{p^3}{16L^2}$, and place it to the right.

Or thus: Erecting a Perpendicular to the Diameter
4 4 (DH), take in it $De = \frac{p}{4}$, placing it to the left hand;
5 and farther thence to the left hand place $ef = \frac{p^3}{16L^2}$;
yea, and farther yet to the left hand must be made
6 6 $fH = \frac{r}{2L^2}$, if in the Equation p and r be affected with the same Signs; but to the right (from the Point f), if p and r are noted with divers Signs.

Then center indeed H, but distance HA, let the Circle (NAM) be described, if it be only a Cubic Equation, that is, if the Quantity S be not had. Fig. 25.

But if S be had, and it be $-S$, then must there farther in this Line HA, both ways produced, be taken on
8 7 the one side $AI = L$, and on the other side $AK = \frac{S}{L^3}$;
9 8 and a Semicircle being described, whose Diameter IK,
10 9 must be erected AL perpendicular to AH, which may meet this Semicircle (ILK) in the Point L.

11 But if $+S$ be had; then moreover in another Semicircle, whose Diameter is HA, must be inscribed
11 10 $AZ = AL$ found. A Circle therefore described pas-
12 sing through L, if it be $-S$, but through Z, if $+S$, will

vel tanget Parabolam, in 1, 2, 3, vel 4 Punctis, à quibus si ad Diametrum demittantur Perpendiculares, obtinebuntur omnes Æquationes radices, tam falsæ, quam veræ. Quarum quidem veræ (ut N O) ad sinistram
14 cadent, si in Æquatione habeatur $-p$; Sed contra, si habeatur ibi $+p$, veræ (ut M O) cadent ad dextram, falsæ verò (ut N O) ad sinistram.

$$\left\{\begin{matrix} 1.\ x^3 - px^2 * + r = 0 \\ 2.\ x^3 + px^2 * - r = 0 \end{matrix}\right. \begin{matrix} \left\{\begin{matrix} 1 \\ 3 \end{matrix}\right\} \begin{matrix} x^4 - px^3 * rx - S = 0 \\ x^4 - px^3 * rx + S = 0 \end{matrix} \\ \left\{\begin{matrix} 2 \\ 4 \end{matrix}\right\} \begin{matrix} x^4 + px^3 * rx - S = 0 \\ x^4 + px^3 * rx + S = 0 \end{matrix} \end{matrix}$$

$$1.\ \text{Caſ. Ubi} \left\{\begin{matrix} -p+r \\ +p-r \end{matrix}\right\};\ \text{\& ſic,} \left\{\begin{matrix} 1 \\ 2 \end{matrix}\right\} \frac{p}{4} + \frac{p^3}{16L^2} \begin{matrix} \sqsupset \\ \sqsubset \end{matrix} \frac{r}{2L^2}.$$

$$1.\ \text{Ubi } \frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}.$$

*Demonstrat.*

| | | |
|---|---|---|
| | | $Ab + bD = b = AD.$ |
| $2+3$ | 11 | $\frac{L}{2} + \frac{p^2}{8L} = b = AD.$ |
| | | $De + ef - fH = d = DH.$ |
| $4+5-6$ | 12 | $\frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = DH.$ |
| 47, e 1 | | $AD^2 + DH^2 = HA^2.$ |
| Q. 11. + Q. 12. | 13 | $b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. *Fig.* 25. |
| | | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2.$ |
| $7 \times 8$ | 14 | $(L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2.$ |
| 47, e 1 | | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad. } In Biquadr. |
| $13+14$ | 15 | $b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. } ſi $-S$. *Fig.* 26. |
| 47, e 1 | | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad. } In Biquadr. |
| $13-14$ | 16 | $b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. } ſi $+S$. *Fig.* 27. |

NO

| | | |
|---|---|---|
| 14 | | will cut or touch the Parabole in 1, 2, 3, or 4 Points; from which, if Perpendiculars be demitted to the Diameter, all the Roots of the Equation will be had, as well false as true: Of which, the true Roots (as NO) will fall to the left, if in the Equation be had — p: But contrarily, if in it be had + p, the true (as MO) will fall to the right, but the false (as NO) to the left. |
| | | $\left\{\begin{array}{l} 1.\ x^3 - px^2 * + r = 0 \left\{\begin{array}{l} 1\ x^4 - px^3 * + rx - S = 0 \\ 3\ x^4 - px^3 * + rx + S = 0 \end{array}\right\} \\ 2.\ x^3 + px^2 * - r = 0 \left\{\begin{array}{l} 2\ x^4 + px^3 * - rx - S = 0 \\ 4\ x^4 + px^3 * - rx + S = 0 \end{array}\right\} \end{array}\right.$ |
| | | *Cas.* 1. Where $\left\{\begin{array}{l} -p+r \\ +p-r \end{array}\right\}$; and so $\left\{\begin{array}{l} 1 \\ 2 \end{array}\right\} \frac{p}{4} + \frac{p^3}{16L^2} \begin{array}{l} \sqsupset \\ \sqsubset \end{array} \frac{r}{2L^2}$. |
| *Supp.* | 17 | $NO = x$. |
| *Supp.* | 17 | $MO = -x$. |
| 17 — 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ |
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| 19 ∽ 11 | 20 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} - \frac{px}{2L} (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} = HP. \end{array}\right.$ |
| ⊕ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. |

he,

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|---|---|---|---|
| 21 | 22 | h [illegible] $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. | |
| 17 — 12 | 23 | $\begin{cases} NO - (PO, u)\, DH = PN. \\ x\,(-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{r}{2L^2} = PN. \end{cases}$ | |
| 17 + 12 | 23 | $\begin{cases} MO + (OP, u)\, DH = PM. \\ -x\,(+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = PM. \end{cases}$ | |
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PN^2$. | |
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PN^2$. | |
| 47, e 1<br>22 + 24 | 25 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.\,Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = Q.\,Rad. \end{cases}$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x^3}$ | 27 | $x^3 - px^2 * + rx = 0$. *Q. e. d.* in Cubic. | *Fig.* 25. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 + rx = S$. | |
| *Transp.* | 30 | $x^4 - px^3 * + rx - S = 0$. *Q. e. d.* in Biquadr. si $-S$. | *Fig.* 26. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 + rx = -S$. | |
| *Transp.* | 33 | $x^4 - px^3 * + rx + S = 0$. *Q. e. d.* in Biquad. si $+S$. | *Fig.* 27. |
| *Supp.* | 17 | $MO = x$. | |
| *Supp.* | 17 | $NO = -x$. | |
| 17 + 1 | 18 | $\begin{cases} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{cases}$ | |

NO

| | | | |
|---|---|---|---|
| 17 — 1 | 18 | $\begin{cases} NO - (OF,u)\, BA = (NF,u)\, OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| 11 ∽ 19 | 20 | $\begin{cases} AD \backsim AO = (DO,u)\, HP. \\ (b,u)\, \frac{L}{2} + \frac{p^3}{8L} - \frac{x^2}{L} - \frac{px}{2L} = HP. \end{cases}$ | |
| $\odot$ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 17 + 12 | 23 | $\begin{cases} MO + (OP,u)\, DH = PM. \\ x\,(+d,u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = PM. \end{cases}$ | |
| 17 — 12 | 23 | $\begin{cases} NO - (OP,u)\, DH = PN. \\ -x\,(-d,u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{r}{2L^2} = PN. \end{cases}$ | |
| $\odot$ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PM^2.$ | |
| $\odot$ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PN^2.$ | |
| 47, *e* 1<br>22 + 24 | 25 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{cases}$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x^3}$ | 27 | $x^3 + px^2 * - rx = 0.$ *Q. e. d.* in Cubic. | *Fig.* 25. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 + px^3 - rx = S.$ | |
| *Transp.* | 30 | $x^4 + px^3 * - rx - S = 0.$ *Q. e. d.* in Biquadr. si — S. | *Fig.* 26. |

| | | |
|---|---|---|
| $25 = 16$ | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 32 | $x^4 + px^3 * - rx = - S$. |
| *Transp.* | 33 | $x^4 + px^3 * - rx + S = 0$. *Q. e. d.* in Biquadr. si $+ S$. *Fig.* 27. |

*Illustrat.*

$$\begin{array}{l} \qquad\quad p. \qquad\qquad r. \\ \left\{\begin{array}{l} x^3 - 16x^2 * + 576 = 0 \\ x^3 - 1.6x^2 * + 0.576 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO = x = 12. \\ no = x = 9.2 \end{array}\right\} \quad MO = -x = -5.21.$$

$$\begin{array}{l} \qquad\quad p. \qquad\qquad r. \\ \left\{\begin{array}{l} x^3 + 16x^2 * - 576 = 0 \\ x^3 + 1.6x^2 * - 0.576 = 0 \end{array}\right\} \end{array}$$

$$MO = x = 5.21. \quad \left\{\begin{array}{l} NO = -x = -12. \\ NO = -x = -9.2. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8L^2} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16L^2} = 0.256. \end{array}\right.$$ *Fig.* 25.

*Central.*

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{l} \frac{p}{4} = 0.4 \\ \frac{p^3}{16L^2} = 0.256 \\ \hline \quad 0.656 \\ -\frac{r}{2L^2} = 0.288 \\ \hline d = 0.368 = DH. \end{array}$$

$$\begin{array}{l} \qquad\quad p. \qquad\qquad r. \qquad\qquad s. \\ \left\{\begin{array}{l} x^4 - 16x^3 * + 900x - 8912 = 0 \\ x^4 - 1.6x^3 * + 0.900x - 0.8912 = 0 \end{array}\right\} \end{array}$$

$NO = x = 14.6 +$ *Fig.* 26.

$MO = -x = -8.8$ *ferè.*

$x^4 +$

$$\left\{\begin{array}{l} \phantom{x^4+}\;p. \qquad\qquad\qquad r. \qquad\qquad\quad s. \\ x^4 + 16x^3 * - 900x - 8912 = 0 \\ x^4 + 1.6x^3 * - 0.900x - 0.8912 = 0 \end{array}\right\}$$

$MO = x = 8.8$ *ferè.*

$NO = -x = -14.6$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

*Central.* Fig.26.

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{r} \frac{p}{4} = 0.4 \\ \frac{p^3}{16L^3} = 0.256 \\ \hline 0.656 \\ -\frac{r}{2L^2} = 0.45 \\ \hline d = 0.206 = DH. \end{array}$$

$$\left\{\begin{array}{l} \phantom{x^4-}\;p. \qquad\qquad\qquad r. \qquad\qquad\quad s. \\ x^4 - 24x^3 * + 1900x + 840 = 0 \\ x^4 - 2.4x^3 * + 1.900x + 0.0840 = 0 \end{array}\right\}$$

$\left\{\begin{array}{l} NO = x = 18. \textit{ ferè.} \\ no = x = 14. \end{array}\right\}$ $MO = -x = -7.6 +$

$$\left\{\begin{array}{l} \phantom{x^4+}\;p. \qquad\qquad\qquad r. \qquad\qquad\quad s. \\ x^4 + 24x^3 * - 1900x + 840 = 0 \\ x^4 + 2.4x^3 * - 1.900x + 0.0840 = 0 \end{array}\right\}$$ Fig.27.

$MO = x = 7.6 +$ $\left\{\begin{array}{l} NO = -x = -18. \textit{ ferè.} \\ no = -x = -14. \end{array}\right.$

*Central.*

$$\frac{L}{2} + \frac{p^2}{8} = b = 1.22 = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = 0.514 = DH$$

2. Ubi $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$.

*Demonstrat.*

| | | | |
|---|---|---|---|
| $2+3$ | 11 | $\begin{cases} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} = b = AD. \end{cases}$ | |
| $6-5-4$ | 12 | $\begin{cases} Hf - fe - eD = d = DH. \\ \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{cases}$ | |
| $Q.11. + Q.12.$ | 13 | $\begin{cases} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =)\ Q.\ \text{Rad. in Cubic.} \end{cases}$ | *Fig.* 28. |
| $7 \times 8$ | 14 | $\begin{cases} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | |
| 47, *e* 1 $13 + 14$ | 15 | $\begin{cases} AH^2 + AL^2 = (HL^2 =)\ Q.\ \text{Rad.} \\ b^2 + d^2 + \frac{S}{L^2} = Q.\ \text{Rad.} \end{cases}$ In Biquadr. si $-S$. | *Fig.* 29. |
| 47, *e* 1 $13 - 14$ | 16 | $\begin{cases} AH^2 - AZ^2 = (HZ^2 =)\ Q.\ \text{Rad.} \\ b^2 + d^2 - \frac{S}{L^2} = Q.\ \text{Rad.} \end{cases}$ In Biquadr. si $+S$. | *Fig.* 30. |
| *Supp.* | 17 | $NO = x.$ | |
| *Supp.* | 17 | $MO = -x.$ | |
| $17 - 1$ | 18 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{cases}$ | |
| $17 + 1$ | 18 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |

AO

| | | | |
|---|---|---|---|
| 19 ∽ 11 | 20 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p2}{8L} = HP. \end{cases}$ | |
| ⊙ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ | |
| 17 + 12 | 23 | $\begin{cases} NO + (OP, u)\ DH = PN. \\ x\ (+d, u) + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ | |
| 17 − 12 | 23 | $\begin{cases} MO - (OP, u)\ DH = PM. \\ -x\ (-d, u) - \frac{r}{2L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^3} = Q.\ Rad. \end{cases}$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 - px^2 * + r = 0.$ *Q. e. d.* in Cubic. | *Fig.* 28. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 + rx = S.$ | |
| *Transp.* | 30 | $x^4 - px^3 * + rx - S = 0.$ *Q. e. d.* in Biquadr. si $+S$. | *Fig.* 29. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 + rx = -S.$ | |
| *Transp.* | 33 | $x^4 - px^3 * + rx + S = 0.$ *Q. e. d.* in Biquad. si $-S$. | *Fig.* 30. |
| *Supp.* | 17 | $MO = x.$ | |
| *Supp.* | 17 | $NO = -x.$ | |

| | | |
|---|---|---|
| $17+1$ | 18 | $\begin{cases} MO + (OF,u)\,BA = (MF,u)\,OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| $17-1$ | 18 | $\begin{cases} NO - (OF,u)\,BA = (NF,u)\,OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 19 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 19 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| $19 \backsim 11$ | 20 | $\begin{cases} AO - AD = (DO,u)\,HP. \\ \frac{x^2}{L} + \frac{px}{2L}\,(-b,u) - \frac{L}{2} - \frac{p^2}{8L} = HP. \end{cases}$ |
| ⊚ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ |
| $17-12$ | 23 | $\begin{cases} MO - (OP,u)\,DH = PM. \\ x\,(-d,u) - \frac{r}{2L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ |
| $17+12$ | 23 | $\begin{cases} NO + (OP,u)\,DH = PN. \\ -x\,(+d,u) + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ |
| ⊚ | 24 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ |
| ⊚ | 24 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ |
| 47, *e* 1<br>$22+24$ | 25 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ \text{Q.Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = \text{Q.Rad.} \end{cases}$ |
| $25 = 13$ | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. |
| $\times \frac{L^2}{x}$ | 27 | $x^3 + px^2 - r = 0$. *Q. e. d.* in Cubic. |

*Fig.* 28.

$\frac{x^4}{L^2}$

| | | | |
|---|---|---|---|
| $25=15$ | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 + px^3 - rx = S.$ | |
| *Transp.* | 30 | $x^4 + px^3 * - rx - S = 0.$ *Q.e.d.* in Biquadr. si $-S$. | *Fig.* 29. |
| $25=16$ | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 + px^3 - rx = -S.$ | |
| *Transp.* | 33 | $x^4 + px^3 * - rx + S = 0.$ *Q.e.d.* in Biquadr. si $+S$. | *Fig.* 30. |

*Illustrat.*

$$\left\{\begin{array}{l} x^3 - \overset{p.}{16}\, x^2 * + \overset{r.}{1536} = 0 \\ x^3 - 1.6\, x^2 * + 1.536 = 0 \end{array}\right\} \quad MO = -x = -8.$$

$$\left\{\begin{array}{l} x^3 + \overset{p.}{16}\, x^2 * - \overset{r.}{1536} = 0 \\ x^3 + 1.6\, x^2 * - 1.536 = 0 \end{array}\right\} \quad MO = x = 8.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512 \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

*Central.* *Fig.* 28.

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{r} \frac{r}{2L^2} = 0.768 \\ \hline -\frac{p^3}{16} = 0.256 \\ -\frac{p}{4} = 0.4 \\ \hline 0.656 \\ d = 0.112 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{16}\, x^3 * + \overset{r.}{1800}\, x - \overset{s.}{19712} = 0 \\ x^4 - 1.6\, x^3 * + 1.800\, x - 1.9712 = 0 \end{array}\right\} \quad \textit{Fig. } 29.$$

$NO = x = 14.$

$MO = -x = -11.4 -$

$x^4 +$

$$\begin{array}{l} p. \qquad\qquad\qquad\qquad r. \qquad\qquad s. \\ \left\{\begin{array}{l} x^4 + 16x^3 * - 1800x - 19712 = 0 \\ x^4 + 1.6x^3 * - 1.800x - 1.9712 = 0 \end{array}\right\} \end{array}$$

MO $= x = 11.4 -$

NO $= - x = - 14.$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

*Central.*

*Fig.29.*

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD \end{array}\right\} \qquad \left.\begin{array}{l} \frac{r}{2L^2} = 0.900 \\ \hline -\frac{p^3}{16} = 0.256 \\ -\frac{p}{4} = 0.4 \\ \hline 0.656 \\ d = 0.244 = DH. \end{array}\right\}$$

$$\begin{array}{l} p. \qquad\qquad r. \qquad\qquad s. \\ \left\{\begin{array}{l} x^4 - 12x^3 * + 2184x + 7232 = 0 \\ x^4 - 1.2x^3 * + 2.184x + 0.7232 = 0 \end{array}\right\} \end{array}$$

MO $= - x = - 8.$

mo $= - x = - 3.6 +$

$$\begin{array}{l} p. \qquad\qquad r. \qquad\qquad s. \\ \left\{\begin{array}{l} x^4 + 12x^3 * - 2184x + 7232 = 0 \\ x^4 + 1.2x^3 * - 2.184x + 0.7232 = 0 \end{array}\right\} \end{array}$$

*Fig.30.*

MO $= x = 8.$

mo $= x = 3.6 +$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array}\right.$$

*Cen-*

*Central.*

$$\left.\begin{array}{l}\frac{L}{2}=0.5\\ \frac{p^2}{8}=0.18\\ \hline b=0.68=AD\\ \hline\end{array}\right\}\begin{array}{l}\frac{r}{2L^2}=1.092\\ \hline +\frac{p^3}{16}=0.108\\ -\frac{p}{4}=0.3\\ \hline 0.408\\ d=0.684=DH.\\ \hline\end{array}$$

29.

$$\left\{\begin{array}{l}3.\ x^3-px^2*-r=0\left\{\begin{array}{l}5\\7\end{array}\right\}\begin{array}{l}x^4-px^3*-rx-S=0\\x^4-px^3*-rx+S=0\end{array}\\ 4.\ x^3+px^2*+r=0\left\{\begin{array}{l}6\\8\end{array}\right\}\begin{array}{l}x^4+px^3*+rx-S=0\\x^4+px^3*+rx+S=0\end{array}\end{array}\right.$$

*Central.*

$$\frac{L}{2}+\frac{p^2}{8L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}=d=DH.$$

2. Caſ. Ubi $\left\{\begin{array}{l}-p-r\\+p+r\end{array}\right\}$

*Demonſtrat.*

30. 2 + 3 | 11 | $\left\{\begin{array}{l}Ab+bD=b=AD.\\ \frac{L}{2}+\frac{p^2}{8L}=b=AD.\end{array}\right.$

4 + 5 + 6 | 12 | $\left\{\begin{array}{l}De+ef+fH=d=DH.\\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}=d=DH.\end{array}\right.$

47, e 1 | 13 | $\left\{\begin{array}{l}AD^2+DH^2=HA^2.\\ b^2+d^2=(HA^2=)\text{Q. Rad. in Cubic.}\end{array}\right.$ *Fig.* 31.

Q.11.+Q.12.

A I x

| | | |
|---|---|---|
| $7 \times 8$ | 14 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2$. <br> $L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2$. |
| 47, e 1 <br> $13 + 14$ | 15 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad. <br> $b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. in Biquadr. si $-$ S. *Fig.* 32. |
| 47, e 1 <br> $13 - 14$ | 16 | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad. <br> $b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. in Biquadr. si $+$ S. *Fig.* 33. |
| *Supp.* | 17 | $NO = x$. |
| *Supp.* | 17 | $MO = -x$. |
| $17 - 1$ | 18 | $NO - (OF, u)\ BA = (NF, u)\ OR$. <br> $x - \frac{p}{2} = OR$. |
| $17 + 1$ | 18 | $MO + (OF, u)\ BA = (MF, u)\ OR$. <br> $-x + \frac{p}{2} = OR$. |
| *Ob para.* | 19 | $L\ .\ NO :: OR\ .\ AO$. <br> $L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO$. |
| *Ob para.* | 19 | $L\ .\ MO :: OR\ .\ AO$. <br> $L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO$. |
| $19 \backsim 11$ | 20 | $AO \backsim AD = (DO, u)\ HP$. <br> $\frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} = HP$. |
| $\odot$ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. |
| 21 | 22 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. |
| $17 - 12$ | 23 | $NO - (OP, u)\ DH = PN$. <br> $x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} = PN$. |
| $17 + 12$ | 23 | $MO + (OP, u)\ DH = PM$. <br> $-x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = PM$. |

| | | | |
|---|---|---|---|
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PN^2.$ | |
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\begin{cases} HP^2 + PN^2 = (HN^2 =) \text{ Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = \text{Q. Rad.} \end{cases}$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 - px^2 * - r = 0$. *Q. e. d.* in Cubic. | *Fig.* 31. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 - rx = S.$ | |
| *Transp.* | 30 | $x^4 - px^3 * - rx - S = 0$. *Q. e. d.* in Biquadr. si $-$ S. | *Fig.* 32. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 - rx = - S.$ | |
| *Transp.* | 33 | $x^4 - px^3 * - rx + S = 0$. *Q. e. d.* in Biquadr. si + S. | *Fig.* 33. |
| *Supp.* | 17 | $MO = x.$ | |
| *Supp.* | 17 | $NO = - x.$ | |
| 17 + 1 | 18 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ | |
| 17 − 1 | 18 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 19 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |

| | | | |
|---|---|---|---|
| 11 ∽ 19 | 20 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\ HP. \\ (b,u)\ \frac{L}{2} + \frac{p^2}{8L} - \frac{x^2}{L} - \frac{px}{2L} = HP. \end{array}\right.$ | |
| $\ominus$ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{px}{2} - \frac{p^2x^2}{4L^2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 17 + 12 | 23 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| 17 − 12 | 23 | $\left\{\begin{array}{l} NO - (OP, u)\ DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| $\ominus$ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PM^2.$ | |
| $\ominus$ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 + px^2 * + rx = 0$. *Q. e. d.* in Cubic. | *Fig*. 31. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 + px^3 + rx = S.$ | |
| *Transp.* | 30 | $x^4 + px^3 * + rx - S = 0$. *Q. e. d.* in Biquadr. si − S. | *Fig*. 32. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 + px^3 + rx = -S.$ | |
| *Transp.* | 33 | $x^4 + px^3 * + rx + S = 0$. *Q. e. d.* in Biquad. si + S. | *Fig*. 33. |

*Illustrat.*

*Illustrat.*

$$\left\{\begin{array}{l} x^3 - \overset{p.}{16} x^2 \ast - \overset{r.}{648} = 0 \\ x^3 - 1.6 x^2 \ast - 0.648 = 0 \end{array}\right\} \quad NO = x = 18.$$

$$\left\{\begin{array}{l} x^3 + \overset{p.}{16} x^2 \ast + \overset{r.}{648} = 0 \\ x^3 + 1.6 x^2 \ast + 0.648 = 0 \end{array}\right\} \quad NO = -x = -18.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512 \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$ *Fig.* 31.

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{r} \frac{p}{4} = 0.4 \\ \frac{p^3}{16} = 0.256 \\ \frac{r}{2} = 0.324 \\ \hline d = 0.980 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{12} x^3 \ast - \overset{r.}{2400} x - \overset{s.}{16000} = 0 \\ x^4 - 1.2 x^3 \ast - 2.400 x - 1.6000 = 0 \end{array}\right\}$$

$NO = x = 20.$
$MO = -x = -5.5-$

*Fig.* 32.

$$\left\{\begin{array}{l} x^4 + \overset{p.}{12} x^3 \ast + \overset{r.}{2400} x - \overset{s.}{16000} = 0 \\ x^4 + 1.2 x^3 \ast + 2.400 x - 1.6000 = 0 \end{array}\right\}$$

$MO = x = 5.5-$
$NO = -x = -20.$

$$\begin{cases} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8L^2} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16L^2} = 0.108. \end{cases}$$

*Central.*

Fig. 32.

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.18 \\ \hline b = 0.68 = AD \end{array}\right\} \begin{array}{l} \frac{p}{4} = 0.3 \\ \frac{p^3}{16} = 0.108 \\ \frac{r}{2} = 1.200 \\ \hline d = 1.608 = DH. \end{array}$$

$$\begin{cases} x^4 - \overset{p.}{12x^3} * - \overset{r.}{500x} + \overset{s.}{6000} = 0 \\ x^4 - 1.2x^3 * - 0.500x + 0.6000 = 0 \end{cases}$$

$NO = x = 12.$
$no = x = 8.$ *ferè.*

Fig. 33.

$$\begin{cases} x^4 + \overset{p.}{12x^3} * + \overset{r.}{500x} + \overset{s.}{6000} = 0 \\ x^4 + 1.2x^3 * + 0.500x + 0.6000 = 0 \end{cases}$$

$NO = -x = -12.$
$no = -x = -8.$ *ferè.*

$$\begin{cases} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{cases}$$

*Central.*

*Central.*

$$\frac{L}{2} = 0.5 \qquad \frac{p^2}{8L} = 0.18 \qquad b = 0.68 = AD$$

$$\frac{p}{4} = 0.3 \qquad \frac{p^3}{16} = 0.108 \qquad \frac{r}{2} = 0.250 \qquad d = 0.658 = DH.$$

*Fig. 33.*

GLAS.

# CLAS. VII.

*De Æquationibus Quadraticis, vel duarum Dimensionum, in quibus nullus deficit Terminorum; & de Quadrato-quadraticis, affectis sub secundo & tertio gradu Parodico; vel de Æquationibus quatuor Dimensionum, ubi deficit quartus Terminus.*

HOrum generum Æquationes ad sequentes formulas reducuntur.

$$\left.\begin{array}{l} 1.\ x^2 - px - q = 0 \left\{\begin{array}{l} 1 \} x^4 - px^3 - qx^2 * - S = 0 \\ 3 \} x^4 - px^3 - qx^2 * + S = 0 \end{array}\right. \\ 2.\ x^2 + px - q = 0 \left\{\begin{array}{l} 2 \} x^4 + px^3 - qx^2 * - S = 0 \\ 4 \} x^4 + px^3 - qx^2 * + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \qquad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH.$$

$$\left.\begin{array}{l} 3.\ x^2 - px + q = 0 \left\{\begin{array}{l} 5 \} x^4 - px^3 + qx^2 * - S = 0 \\ 7 \} x^4 - px^3 + qx^2 * + S = 0 \end{array}\right. \\ 4.\ x^2 + px + q = 0 \left\{\begin{array}{l} 6 \} x^4 + px^3 + qx^2 * - S = 0 \\ 8 \} x^4 + px^3 + qx^2 * + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \qquad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} = d = DH.$$

*Reg. Gen.*

Describatur itaque Parabola (NAM), cujus Latus Rectum sit L (seu 1), & Axis (ay); ad quem
1 ordinatìm applicetur $BA = \frac{p}{2}$, occurrens Parabolæ in B & A: Ex puncto A (puta) ducatur Diameter,
2 vel Axi parallela (Ay); in quâ sumendo $Ab = \frac{L}{2}$, & bc =

## CLAS. VII.

*Of Quadratics, or of Equations of two Dimensions, in which neither of the Terms is wanting; and of Quadrato-quadratics, affected under the second and third Parodic Degree; or of Equations of four Dimensions, where the fourth Term is deficient.*

ALL Equations of both these kinds are reduced to these following forms.

$$\begin{cases} 1.\ x^2 - px - q = 0 \begin{cases} 1 \ x^4 - px^3 - qx^2 * - S = 0 \\ 3 \ x^4 - px^3 - qx^2 * + S = 0 \end{cases} \\ 2.\ x^2 + px - q = 0 \begin{cases} 2 \ x^4 + px^3 - qx^2 * - S = 0 \\ 4 \ x^4 + px^3 - qx^2 * + S = 0 \end{cases} \end{cases}$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH.$$

$$\begin{cases} 3.\ x^2 - px + q = 0 \begin{cases} 5 \ x^4 - px^3 + qx^2 * - S = 0 \\ 7 \ x^4 - px^3 + qx^2 * + S = 0 \end{cases} \\ 4.\ x^2 + px + q = 0 \begin{cases} 6 \ x^4 + px^3 + qx^2 * - S = 0 \\ 8 \ x^4 + px^3 + qx^2 * + S = 0 \end{cases} \end{cases}$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} = d = DH.$$

*Gen. Rule* Let a Parabole (NAM) therefore be described, whose *Latus Rectum* L (or 1), and Axe (a y); to
1 1 which, ordinately apply $BA = \frac{p}{2}$, meeting the Parabole in B and A: From the Point (suppose) A, draw the Diameter, or a Parallel to the Axe
2 (*viz.* A y); in which, taking $Ab = \frac{L}{2}$, and

bc =

2 3 4 $bc = \frac{p^2}{8L}$, oportet facere $cD = \frac{q}{2L}$; eamque sumere
in Diametro continuatâ versus y, si in Æquatione habeatur $-q$; sed versus alteram partem (sursum,) si habeatur ibi $+q$.

3 Porrò, è Puncto D, (erigendo ad Diametrum per-
5 pendicularem DH,) oportet in eâ sumere $De = \frac{p}{4}$,
4 6 7 & $ef = \frac{p^3}{16L^2}$; imo, & $fH = \frac{pq}{4L^2}$, si in Æquatione ha-
beatur $-q$ (collocanda ad sinistram, &c.) Quod si ha-
5 7 beatur ibi $+q$; tum $fH = \frac{pq}{4L^2}$, ad dextram est collo-
canda, à Puncto f. Tum Centro quidem H, inter-
6 8 vallo verò H A, describatur Circulus (N A M), si
Æquatio tantùm Quadratica fuerit, hoc est, si non
habeatur Quantitas S.

7 9 Ast si habeatur S, & signo quidem $-$ adfecta (nempe
$-S$), oportet ulteriùs in hac lineâ A H, utrinque
8 10 productâ, sumere $AI = L$, ex unâ parte, & ex alterâ
9 11 $AK = \frac{S}{L^3}$; descriptoque Semicirculo, cujus Diameter
10 12 I K, erigere A L perpendicularem ad A H, quæ occur-
rat huic Semicirculo (I L K) in puncto L.
11 13 Quòd si verò habeatur $+S$; oportet insuper in
alio Semicirculo, cujus Diameter sit A H, inscribere
11 14 $AZ = AL$ inventæ.

12 Circulus igitur descriptus, transiens per L, si sit $-S$; per Z verò, si sit $+S$, secabit vel tanget Parabolam, in tot Punctis, quot Æquatio diversas admittet Radices; è quibus si ad Diametrum demittantur Perpendiculares, obtinebuntur omnes Æquationis radices, tàm falsæ, quàm veræ. Quarum quidem veræ (ut N O) ad sinistram cadent, & falsæ (ut M O) ad dextram
14 Diametri, si in Æquatione habeatur $-p$: Sed contra,
15 si habeatur ibi $+p$, veræ (ut M O) cadent ad dextram, falsæ verò (ut N O) ad sinistram.

1. $x^3$

2 3 4 $bc = \frac{p^2}{8L}$, must be made also $cD = \frac{q}{2L}$, and be pla-
ced in the same Diameter continued towards y, if in
the Equation be had $-q$; but towards the other part
(upward,) if be had there $+q$.

3 Moreover, from the Point D, (DH being ere-
cted perpendicular to the Diameter) must be made
5 5 6 $De = \frac{p}{4}$, and $ef = \frac{p^3}{16L^2}$; nay, and $fH = \frac{pq}{4L^2}$ (far-
7 ther to the left hand), if in the Equation be had $-q$;
7 but if $+q$, then from the Point f, $fH = \frac{pq}{4L^2}$ is to be
placed to the right hand. Then center truly H, but
6 8 distance HA, let a Circle (NAM) be described, if
it be only a Quadratic Equation, that is, if the Quan-
tity S be not had.

7 9 But if S be had, and it be $-S$, then farther in this
Line AH, both ways produced, must be taken on the
8 10 9 11 one side $AI = L$, and on the other side $AK = \frac{S}{L^3}$;
and a Semicircle being described, whose Diameter IK,
10 12 must be erected AL perpendicular to AH, which may
meet this Semicircle (ILK) in the Point L.
11 13 But if $+S$ be had, there must moreover in ano-
ther Semicircle, whose Diameter is AH, be inscribed
11 14 $AZ = AL$ found.

12 A Circle therefore described, passing through L, if
it be $-S$, but through Z, if it be $+S$, will cut or
touch the Parabole in as many Points, as the Equation
will admit Roots; from which, if Perpendiculars be
demitted to the Diameter, all the Roots of the Equa-
tion, as well true as false, will be had: Of which, the
true (as NO) will fall to the left hand, and the false
14 (as MO) to the right side of the Diameter, if in the
15 Equation be had $-p$: But contrarily, if be had there
$+p$, the true (as MO) will fall to the right, but
the false (as NO) to the left.

$$\left\{\begin{array}{l} 1.\ x^2 - px - q = 0 \\ 2.\ x^2 + px - q = 0 \end{array}\right. \quad \begin{array}{l} 1 \{ x^4 - px^3 - qx^2 * - S = 0 \} \\ 3 \{ x^4 - px^3 - qx^2 * + S = 0 \} \\ 2 \{ x^4 + px^3 - qx^2 * - S = 0 \} \\ 4 \{ x^4 + px^3 - qx^2 * + S = 0 \} \end{array}$$

*Demonstrat.*

| | | | | |
|---|---|---|---|---|
| 2+3+4 | 15 | $\left\{\begin{array}{l} Ab + bc + cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \end{array}\right\}$ | 1°. Ubi $-q$. | |
| 5+6+7 | 16 | $\left\{\begin{array}{l} De + ef + fH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH. \end{array}\right.$ | | |
| 47, e 1 <br> Q.15. + Q.16. | 17 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Quadrat.} \end{array}\right.$ | | *Fig.* 34. |
| 10 × 11 | 18 | $\left\{\begin{array}{l} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$ | | |
| 47, e 1 <br> 17 + 18 | 19 | $\left\{\begin{array}{l} AH^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{ Q.Rad.} \end{array}\right.$ | $\left.\begin{array}{l}\text{In Biquadr.} \\ \text{si} - S.\end{array}\right\}$ | *Fig.* 35. |
| 47, e 1 <br> 17 − 18 | 20 | $\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{ Q.Rad.} \end{array}\right.$ | $\left.\begin{array}{l}\text{In Biquadr.} \\ \text{si} + S.\end{array}\right\}$ | *Fig.* 36. |
| *Supp.* | 21 | $NO = x.$ | | |
| *Supp.* | 21 | $MO = -x.$ | | |
| 21 − 1 | 22 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ | | |
| 21 + 1 | 22 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ | | |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | | |

L . MO

| | | | |
|---|---|---|---|
| *Ob para.* | 23 | $\begin{cases} L \;.\; MO :: OR \;.\; AO. \\ L \;.\; -x :: -x+\frac{p}{2} \;.\; \frac{x^2}{L}-\frac{px}{2L}=AO. \end{cases}$ | |
| $23 \backsim 15$ | 24 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L}-\frac{px}{2L}\ (-b, u) -\frac{L}{2}-\frac{p2}{8L}-\frac{q}{2L}=HP. \end{cases}$ | |
| $\circledcirc$ | 25 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{p^2x^2}{L^2}-x^2-\frac{p^2x^2}{4L^2}-\frac{qx^2}{L^2}+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| 25 | 26 | he, $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}-x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| $21-16$ | 27 | $\begin{cases} NO-(OP, u)\ DH = PN. \\ x\ (-d, u) -\frac{p}{4}-\frac{p^3}{16L^2}-\frac{pq}{4L^2}=PN. \end{cases}$ | |
| $21+16$ | 27 | $\begin{cases} MO+(OP, u)\ DH = PM. \\ -x\ (+d, u) +\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}=PM. \end{cases}$ | |
| $\circledcirc$ | 28 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=PN^2.$ | |
| $\circledcirc$ | 28 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=PM^2.$ | |
| 47, *e* 1<br>$26+28$ | 29 | $\begin{cases} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=(HN^2=)\ Q.\,Rad. \end{cases}$ | |
| $29=17$ | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x^2}$ | 31 | $x^2-px-q=0$. *Q. e. d.* in Quadratic. | *Fig.* 34. |
| $29=19$ | 32 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4-px^3-qx^2=S.$ | |
| *Transp.* | 34 | $x^4-px^3-qx^2 * -S=0.$ In Biquadr. si $-S$. | *Fig.* 35. |
| $29=20$ | 35 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4-px^3-qx^2=-S.$ | |
| *Transp.* | 37 | $x^4-px^3-qx^2 * +S=0.$ *Q. e. d.* in Biquadr. si $+S$. | *Fig.* 36. |

| | | |
|---|---|---|
| *Supp.* | 21 | $MO = x.$ |
| *Supp.* | 21 | $NO = -x.$ |
| $21+1$ | 22 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| $21-1$ | 22 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| $23-15$ | 24 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\, (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ |
| $\ominus$ | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{qx^2}{L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$ |
| 25 | 26 | $he,\ b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ |
| $21+16$ | 27 | $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ x\, (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PM. \end{array}\right.$ |
| $21-16$ | 27 | $\left\{\begin{array}{l} NO - (OP, u)\, DH = PN. \\ -x\, (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PN. \end{array}\right.$ |
| $\ominus$ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = PM^2.$ |
| $\ominus$ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = PN^2.$ |
| 47, e 1<br>$26+28$ | 29 | $\left\{\begin{array}{l} HP^2 + PM^2 = HM^2 = Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = Q.Rad. \end{array}\right.$ |

$\frac{x^4}{L^2}$

| | | | |
|---|---|---|---|
| $29 = 17$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = 0$; in $\frac{x^2}{L^2}$. | |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px - q = 0$. *Q. e. d.* in Quadratic. | *Fig.* 34. |
| $29 = 19$ | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4 + px^3 - qx^2 = S$. | |
| *Transp.* | 34 | $x^4 + px^3 - qx^2 * - S = 0$. *Q. e. d.* in Biquad. si $-S$. | *Fig.* 35. |
| $29 = 20$ | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4 + px^3 - qx^2 = -S$. | |
| *Transp.* | 37 | $x^4 + px^3 - qx^2 * + S = 0$. *Q. e. d.* in Biquad. si $+S$. | *Fig.* 36. |

*Illustrat.*

$$\begin{Bmatrix} x^2 - \overset{p.}{12}x - \overset{q.}{64} = 0 \\ x^2 - 1.2x - 0.64 = 0 \end{Bmatrix} \quad \begin{matrix} NO = x = 16. \\ MO = -x = -4. \end{matrix}$$

$$\begin{Bmatrix} x^2 + \overset{p.}{12}x + \overset{q.}{64} = 0 \\ x^2 + 1.2x + 0.64 = 0 \end{Bmatrix} \quad \begin{matrix} MO = x = 4. \\ NO = -x = -16. \end{matrix}$$

*Fig.* 34.

*Central.*

$$\frac{p}{4} = 0.3. \qquad \frac{p^2}{8} = 0.18. \qquad \frac{p^3}{16} = 0.108.$$

$$\frac{L}{2} + \frac{p^2}{8} + \frac{q}{2L} = 1 = b. \qquad \frac{p}{4} + \frac{p^3}{16} + \frac{pq}{4} = 0.600 = d.$$

$$\begin{Bmatrix} x^4 - \overset{p.}{8}x^3 - \overset{q.}{40}x^2 * - \overset{s.}{8624} = 0 \\ x^4 - 0.8x^3 - 0.40x^2 * - 0.8624 = 0 \end{Bmatrix}$$

$NO = x = 14.$
$MO = -x = -8.8+$

*Fig.* 35.

$x^4 +$

$$\begin{Bmatrix} x^4 + \overset{p.}{8}x^3 - \overset{q.}{40}x^2 * - \overset{s.}{8624} = 0 \\ x^4 + 0.8x^3 - 0.40x^2 * - 0.8624 = 0 \end{Bmatrix}$$

MO = x = 8.8 +
NO = − x = − 14.

$$\begin{Bmatrix} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 0.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{Bmatrix}$$

Fig. 35.

*Central.*

$$\left.\begin{aligned} \frac{L}{2} &= 0.5 \\ \frac{p^2}{8} &= 0.08 \\ \frac{q}{2} &= 0.20 \\ \hline b &= 0.78 = AD \end{aligned}\right\} \qquad \begin{aligned} \frac{p}{4} &= 0.2 \\ \frac{p^3}{16} &= 0.032 \\ \frac{pq}{4} &= 0.080 \\ \hline d &= 0.312 = DH. \end{aligned}$$

$$\begin{Bmatrix} x^4 - \overset{p.}{8}x^3 - \overset{q.}{78}x^2 * + \overset{s.}{4320} = 0 \\ x^4 - 0.8x^3 - 0.78x^2 * + 0.4320 = 0 \end{Bmatrix}$$

NO = x = 12.
no = x = 7.1 +

$$\begin{Bmatrix} x^4 + \overset{p.}{8}x^3 - \overset{q.}{78}x^2 * + \overset{s.}{4320} = 0 \\ x^4 + 0.8x^3 - 0.78x^2 * + 0.4320 = 0 \end{Bmatrix}$$

Fig. 36.

NO = − x = − 12.
no = − x = − 7.1 +

$$\begin{Bmatrix} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 0.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{Bmatrix}$$

*Central.*

*Central.*

$$\left.\begin{array}{l}\frac{L}{2}=0.5\\ \frac{p^2}{8}=0.08\\ \frac{q}{2}=0.39\\ \hline b=0.97=AD\end{array}\right\} \quad \left.\begin{array}{l}\frac{p}{4}=0.2\\ \frac{p^3}{16}=0.032\\ \frac{pq}{4}=0.156\\ \hline d=0.388=DH.\end{array}\right.$$ Fig. 36.

$$\left\{\begin{array}{l}3.\ x^2-px+q=0\left\{\begin{array}{l}5\ \{x^4-px^3+qx^2 * -S=0\}\\ 7\ \{x^4-px^3+qx^2 * +S=0\}\end{array}\right.\\ 4.\ x^2+px+q=0\left\{\begin{array}{l}6\ \{x^4+px^3+qx^2 * -S=0\}\\ 8\ \{x^4+px^3+qx^2 * +S=0\}\end{array}\right.\end{array}\right.$$

*Cas.* 1. Ubi $\frac{L}{2}+\frac{p^2}{8L}\sqsupset\frac{q}{2L}$.

*Demonstrat.*

2+3−4 | 15 | $\left\{\begin{array}{l}Ab+bc-cD=b=AD.\\ \frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=b=AD.\end{array}\right.$

5+6−7 | 16 | $\left\{\begin{array}{l}De+ef-fH=d=DH.\\ \frac{p}{4}+\frac{p^3}{16L^2}-\frac{pq}{4L^2}=d=DH.\end{array}\right.$

47, e 1 Q.15.+ Q.16. | 17 | $\left\{\begin{array}{l}AD^2+DH^2=HA^2.\\ b^2+d^2=(HA^2=)\ \text{Q. Rad. in Quadr.}\end{array}\right.$ Fig. 37.

10 × 11 | 18 | $\left\{\begin{array}{l}AI\times AK=\ \text{(ob Circl.)}\ AL^2=\ \text{(per constr.)}\ AZ^2.\\ L\times\frac{S}{L^3}=)\ \frac{S}{L^2}=AL^2=AZ^2.\end{array}\right.$

47, e 1 17+18 | 19 | $\left\{\begin{array}{l}AH^2+AL^2=HL^2.\\ b^2+d^2+\frac{S}{L^2}=(HL^2=)\ \text{Q. Rad.}\end{array}\right\}$ $\left\{\begin{array}{l}\text{In Biquadr.}\\ \text{si}-S.\end{array}\right\}$ Fig. 38.

AH²

| | | |
|---|---|---|
| 47, e 1<br>17 — 18 | 20 | $\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)\ Q.Rad. \end{array}\right\}$ In Biquadr. ss + S. } *Fig.* 39. |
| *Supp.* | 21 | $NO = x.$ |
| *Supp.* | 21 | $MO = -x.$ |
| 21 — 1 | 22 | $\left\{\begin{array}{l} NO - (OF,\ u)\ BA = (NF,\ u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ |
| 21 + 1 | 22 | $\left\{\begin{array}{l} MO + (OF,\ u)\ BA = (MF,\ u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| 23 ∽ 15 | 24 | $\left\{\begin{array}{l} AO \backsim AD = (DO,\ u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b,\ u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊚ | 25 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{L^2} + \frac{qx^2}{L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ |
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ |
| 21 — 16 | 27 | $\left\{\begin{array}{l} NO - (OP,\ u)\ DH = PN. \\ x\ (-d,\ u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PN. \end{array}\right.$ |
| 21 + 16 | 27 | $\left\{\begin{array}{l} MO + (OP,\ u)\ DH = PM. \\ -x\ (+d,\ u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PM. \end{array}\right.$ |
| ⊚ | 28 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = PN^2.$ |
| ⊚ | 28 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{L^2} = PM^2.$ |

| | | | |
|---|---|---|---|
| 47, *e* 1<br>26 + 28 | 29 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = (HN^2 =) \text{ Q. Rad.} \end{cases}$ | |
| 29 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 - px + q = 0$. *Q. e. d.* in Quadratic. | *Fig.* 37. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4 - px^3 + qx^2 = S$. | |
| *Transp.* | 34 | $x^4 - px^3 + qx^2 * - S = 0$. In Biquadr. si $- S$. | *Fig.* 38. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4 - px^3 + qx^2 = - S$. | |
| *Transp.* | 37 | $x^4 - px^3 + qx^2 * + S = 0$. *Q. e. d.* in Biquadr. si $+ S$. | *Fig.* 39. |
| *Supp.* | 21 | $MO = x$. | |
| *Supp.* | 21 | $NO = - x$. | |
| 21 + 1 | 22 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ | |
| 21 − 1 | 22 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ - x - \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 23 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 23 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, - x :: - x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| 23 ∽ 15 | 24 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{cases}$ | |
| G | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{qx^2}{L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ | |

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he,

| | | | |
|---|---|---|---|
| 25. | 26 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$. | |
| 21 + 16 | 27 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ x\,(+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PM. \end{cases}$ | |
| 21 − 16 | 27 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ -x\,(- d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PN. \end{cases}$ | |
| ⊙ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = PM^2$. | |
| ⊙ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = PN^2$. | |
| 47, e 1<br>26 + 28 | 29 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = \text{Q. Rad.} \end{cases}$ | |
| 29 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px + q = 0$. *Q. e. d.* in Quadratic. | *Fig.* 37. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4 + px^3 + qx^2 = S$. | |
| *Transp.* | 34 | $x^4 + px^3 + qx^2 * - S = 0$. *Q. e. d.* in Biquad. si $-S$. | *Fig.* 38. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4 + px^3 + qx^2 = -S$. | |
| *Transp.* | 37 | $x^4 + px^3 + qx^2 * + S = 0$. *Q. e. d.* in Biquad. si $+S$. | *Fig.* 39. |

*Illustrat.*

$$3\ \begin{cases} x^2 - \overset{p.}{28}\,x + \overset{q.}{180} = 0 \\ x^2 - 2.8\,x + 1.80 = 0 \end{cases} \qquad \begin{matrix} NO = x = 18. \\ no = x = 10. \end{matrix}$$

*Fig.* 37.

$$4\ \begin{cases} x^2 + \overset{p.}{28}\,x + \overset{q.}{180} = 0 \\ x^2 + 2.8\,x + 1.80 = 0 \end{cases} \qquad \begin{matrix} NO = -x = -18. \\ no = -x = -10. \end{matrix}$$

$$\begin{cases} \frac{p}{2} = 1.4. & \frac{p^2}{4} = 1.96. & \frac{p^3}{8} = 2.744. \\ \frac{p}{4} = 0.7. & \frac{p^2}{8} = 0.98. & \frac{p^3}{16} = 1.372. \end{cases}$$

*Central.*

Fig. 37.

$$\begin{array}{rl} \frac{L}{2} & = 0.5 \\ \frac{p^2}{8} & = 0.98 \\ \hline & 1.48 \\ -\frac{q}{2} & = 0.90 \\ \hline b & = 0.58 = AD. \\ \hline \end{array} \qquad \begin{array}{rl} \frac{p}{4} & = 0.7 \\ \frac{p^3}{16} & = 1.372 \\ \hline & 2.072 \\ -\frac{pq}{4} & = 1.260 \\ \hline d & = 0.812 = DH. \\ \hline \end{array}$$

$$5 \begin{cases} x^4 - \overset{p.}{28}\, x^3 + \overset{q.}{240}\, x^2 * - \overset{s.}{6000} = 0 \\ x^4 - 2.8\, x^3 + 2.40\, x^2 * - 0.6000 = 0 \end{cases}$$

$NO = x = 10.$

$MO = -x = - 4.0+$

Fig. 38.

$$6 \begin{cases} x^4 + \overset{p.}{28}\, x^3 + \overset{q.}{240}\, x^2 * - \overset{s.}{6000} = 0 \\ x^4 + 2.8\, x^3 + 2.40\, x^2 * - 0.6000 = 0 \end{cases}$$

$NO = -x = -10.$

$MO = +x = 4.0+$

$$\begin{cases} \frac{p}{2} = 1.4. & \frac{p^2}{4} = 1.96. & \frac{p^3}{8} = 2.744. \\ \frac{p}{4} = 0.7. & \frac{p^2}{8} = 0.98. & \frac{p^3}{16} = 1.372. \end{cases}$$

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.98 \\ \hline 1.48 \\ -\frac{q}{2} = 1.20 \\ \hline b = 0.28 = AD \\ \hline \end{array}\right\} \quad \begin{array}{r} \frac{p}{4} = 0.7 \\ \frac{p^3}{16} = 1.372 \\ \hline 2.072 \\ -\frac{pq}{4} = 1.680 \\ \hline d = 0.392 = DH. \\ \hline \end{array}$$

*Fig.* 38.

$$7 \left\{\begin{array}{l} \overset{p.}{} \qquad \overset{q.}{} \qquad \overset{s.}{} \\ x^4 - 28x^3 + 90x^2 * + 26112 = 0 \\ x^4 - 2.8x^3 + 0.90x^2 * + 2.6112 = 0 \end{array}\right\}$$

$NO = x = 20. +$

$no = x = 16.$

$$8 \left\{\begin{array}{l} \overset{p.}{} \qquad \overset{q.}{} \qquad \overset{s.}{} \\ x^4 + 28x^3 + 90x^2 * + 26112 = 0 \\ x^4 + 2.8x^3 + 0.90x^2 * + 2.6112 = 0 \end{array}\right\}$$

$NO = -x = -20. +$

$no = -x = -16.$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.4. & \frac{p^2}{4} = 1.96. & \frac{p^3}{8} = 2.744. \\ \frac{p}{4} = 0.7. & \frac{p^2}{8} = 0.98. & \frac{p^3}{16} = 1.372. \end{array}\right.$$

*Fig.* 39.

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.98 \\ \hline 1.48 \\ -\frac{q}{2} = 0.45 \\ \hline b = 1.03 = AD \\ \hline \end{array}\right\} \quad \begin{array}{r} \frac{p}{4} = 0.7 \\ \frac{p^3}{16} = 1.372 \\ \hline 2.072 \\ -\frac{pq}{4} = 0.630 \\ \hline d = 1.442 = DH. \\ \hline \end{array}$$

*Cas.* 2.

*Cas.* 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$.

*Demonstrat.*

| | | | |
|---|---|---|---|
| 4—3—2 | 15 | $\begin{cases} cD - bc - Ab = b = AD. \\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD. \end{cases}$ | |
| 7—6—5 | 16 | $\begin{cases} fH - ef - De = d = DH. \\ \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{cases}$ | |
| 47, e 1 Q.15.+ Q.16. | 17 | $\begin{cases} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Quadr.} \end{cases}$ | *Impossib.* |
| 10 × 11 | 18 | $\begin{cases} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | |
| 47, e 1 17 + 18 | 19 | $\begin{cases} AH^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{ Q. Rad.} \end{cases}$ $\begin{Bmatrix} \text{In Biquadr.} \\ \text{si} - S. \end{Bmatrix}$ | *Fig.* 40. |
| 47, e 1 17 — 18 | 20 | $\begin{cases} AH^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{ Q. Rad.} \end{cases}$ $\begin{Bmatrix} \text{In Biquadr.} \\ \text{si} + S. \end{Bmatrix}$ | *Impossib.* |
| *Supp.* | 21 | $NO = x.$ | |
| *Supp.* | 21 | $MO = -x.$ | |
| 21 — 1 | 22 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{cases}$ | |
| 21 + 1 | 22 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 23 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |

L . MO

| | | |
|---|---|---|
| *Ob para.* | 23 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ |
| 23 + 15 | 24 | $\begin{cases} AO + AD = (DO,\ u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (+ b,\ u) + \frac{q}{2L^2} - \frac{p^2}{8L} - \frac{L}{2} = HP. \end{cases}$ |
| ⊙ | 25 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} + \frac{qx^2}{L^2} - \frac{p^2x^2}{4L^2} - x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = HP^2.$ |
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = HP^2.$ |
| 21 + 16 | 27 | $\begin{cases} NO + (OP,\ u)\ DH = PN. \\ x\ (+ d,\ u) + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ |
| 21 — 16 | 27 | $\begin{cases} MO - (OP,\ u)\ DH = PM. \\ -x\ (- d,\ u) - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ |
| ⊙ | 28 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ |
| ⊙ | 28 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ |
| 47, e 1<br>26 + 28 | 29 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = (HN^2 =)\ \text{Q. Rad.} \end{cases}$ |
| 29 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. |
| $\times \frac{L^2}{x^2}$ | 31 | $x^3 - px + q = 0$. *Q. e. d.* in Quadr. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 33 | $x^4 - px^3 + qx^2 = S.$ |
| *Transp.* | 34 | $x^4 - px^3 + qx^2 * - S = 0$. *Q. e. d.* in Biquad. si $- S$. *Fig.* 40. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 36 | $x^4 - px^3 + qx^2 = - S.$ |
| *Transp.* | 37 | $x^4 - px^3 + qx^2 * + S = 0$. *Q. e. d.* in Biquadr. si $+ S$. |

MO

| | | |
|---|---|---|
| *Supp.* | 21 | $MO = x.$ |
| *Supp.* | 21 | $NO = -x.$ |
| $21 + 1$ | 22 | $\begin{cases} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| $21 - 1$ | 22 | $\begin{cases} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 23 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 23 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| $23 + 15$ | 24 | $\begin{cases} AO + AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\, (+b, u) + \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = HP. \end{cases}$ |
| $\odot$ | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} + \frac{qx^2}{L^2} - \frac{p^2x^2}{4L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2.$ |
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2.$ |
| $21 - 16$ | 27 | $\begin{cases} MO - (OP, u)\, DH = PM. \\ x\, (-d, u) - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ |
| $21 + 16$ | 27 | $\begin{cases} NO + (OP, u)\, DH = PN. \\ -x\, (+d, u) + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ |
| $\odot$ | 28 | $d^2 + x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ |
| $\odot$ | 28 | $d^2 + x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ |
| 47, *e* 1<br>$26 + 28$ | 29 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = (HM^2 =)\ Q,\ Rad. \end{cases}$ |

| | | |
|---|---|---|
| $29 = 17$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px + q = 0$. *Q.e.d.* in Quadratic. |
| $29 = 19$ | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 33 | $x^4 + px^3 + qx^2 = S$. |
| *Transp.* | 34 | $x^4 + px^3 + qx^2 * - S = 0$. *Q.e.d.* in Biquadr. si $-S$. *Fig.* 40. |
| $29 = 20$ | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 36 | $x^4 + px^3 + qx^2 = -S$. |
| *Transp.* | 37 | $x^4 + px^3 + qx^2 * + S = 0$. *Q.e.d.* in Biquadr. si $+S$. |

*Illustrat.*

$$5 \left\{ \begin{array}{l} x^4 - \overset{p.}{16}\, x^3 + \overset{q.}{212}\, x^2 * - \overset{s.}{23616} = 0 \\ x^4 - 1.6\, x^3 + 2.12\, x^2 * - 2.3616 = 0 \end{array} \right\}$$

NO $= x = 12$.
MO $= -x = -7.+$

$$6 \left\{ \begin{array}{l} x^4 + \overset{p.}{16}\, x^3 + \overset{q.}{212}\, x^2 * - \overset{s.}{23616} = 0 \\ x^4 + 1.6\, x^3 + 2.12\, x^2 * - 2.3616 = 0 \end{array} \right\} \quad \textit{Fig.}\ 40.$$

MO $= x = 7.+$
NO $= -x = -12$.

$$\left\{ \begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array} \right.$$

Central.

*Central.*

$$+\frac{q}{2} = 1.06 \qquad +\frac{pq}{4} = 0.848$$

$$-\frac{p^2}{8} = 0.32 \qquad -\frac{p^3}{16} = 0.256$$

$$-\frac{L}{2} = 0.5 \qquad -\frac{p}{4} = 0.4$$

$$0.82 \qquad 0.656$$

$$b = 0.24 = AD. \qquad d = 0.192 = DH.$$

P CLAS.

## CLAS. VIII.

*De Æquationibus Cubicis, & Quadrato-quadraticis, sub omnibus gradibus Parodicis affectis; vel, de Æquationibus trium & quatuor Dimensionum, in quibus nullus deficit Terminorum.*

Hujus quidem census Æquationes ad octo, illius verò ad sexdecem formulas reduci possint.

$$\left\{\begin{array}{l} 1.\ x^3 - px^2 - qx - r = 0 \left\{\begin{array}{l} 1\ \}\ x^4 - px^3 - qx^2 - rx - S = 0 \\ 3\ \}\ x^4 - px^3 - qx^2 - rx + S = 0 \end{array}\right. \\ 2.\ x^3 + px^2 - qx + r = 0 \left\{\begin{array}{l} 2\ \}\ x^4 + px^3 - qx^2 + rx - S = 0 \\ 4\ \}\ x^4 + px^3 - qx^2 + rx + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 - qx + r = 0 \left\{\begin{array}{l} 5\ \}\ x^4 - px^3 - qx^2 + rx - S = 0 \\ 7\ \}\ x^4 - px^3 - qx^2 + rx + S = 0 \end{array}\right. \\ 4.\ x^3 + px^2 - qx - r = 0 \left\{\begin{array}{l} 6\ \}\ x^4 + px^3 - qx^2 - rx - S = 0 \\ 8\ \}\ x^4 + px^3 - qx^2 - rx + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 5.\ x^3 - px^2 + qx + r = 0 \left\{\begin{array}{l} 9\ \}\ x^4 - px^3 + qx^2 + rx - S = 0 \\ 11\ \}\ x^4 - px^3 + qx^2 + rx + S = 0 \end{array}\right. \\ 6.\ x^3 + px^2 + qx - r = 0 \left\{\begin{array}{l} 10\ \}\ x^4 + px^3 + qx^2 - rx - S = 0 \\ 12\ \}\ x^4 + px^3 + qx^2 - rx + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH.$$

7. $x^3$

# CLAS. VIII.

## *Of Cubic and Biquadratic Equations, affected under all their Parodic Degrees; or, of Equations of three and four Dimensions, in which neither of their Terms is wanting.*

ALL Cubic Equations of this kind may be reduced to eight, but Quadrato-quadratics to sixteen forms.

$$\left\{\begin{array}{l} 1.\ x^3-px^2-qx-r=0 \left\{\begin{array}{l} 1\ \left\{ x^4-px^3-qx^2-rx-S=0 \right\} \\ 3\ \left\{ x^4-px^3-qx^2-rx+S=0 \right\} \end{array}\right. \\ 2.\ x^3+px^2-qx+r=0 \left\{\begin{array}{l} 2\ \left\{ x^4+px^3-qx^2+rx-S=0 \right\} \\ 4\ \left\{ x^4+px^3-qx^2+rx+S=0 \right\} \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH.$$

$$\left\{\begin{array}{l} 3.\ x^3-px^2-qx+r=0 \left\{\begin{array}{l} 5\ \left\{ x^4-px^3-qx^2+rx-S=0 \right\} \\ 7\ \left\{ x^4-px^3-qx^2+rx+S=0 \right\} \end{array}\right. \\ 4.\ x^3+px^2-qx-r=0 \left\{\begin{array}{l} 6\ \left\{ x^4+px^3-qx^2-rx-S=0 \right\} \\ 8\ \left\{ x^4+px^3-qx^2-rx+S=0 \right\} \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}\backsim\frac{r}{2L^2}=d=DH.$$

$$\left\{\begin{array}{l} 5.\ x^3-px^2+qx+r=0 \left\{\begin{array}{l} 9\ \left\{ x^4-px^3+qx^2+rx-S=0 \right\} \\ 11\ \left\{ x^4-px^3+qx^2+rx+S=0 \right\} \end{array}\right. \\ 6.\ x^3+px^2+qx-r=0 \left\{\begin{array}{l} 10\ \left\{ x^4+px^3+qx^2-rx-S=0 \right\} \\ 12\ \left\{ x^4+px^3+qx^2-rx+S=0 \right\} \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}\backsim\frac{q}{2L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}\backsim\frac{pq}{4L^2}\backsim\frac{r}{2L^2}=d=DH.$$

$$\left\{\begin{array}{l} 7.\ x^3-px^2+qx-r=0 \left\{\begin{array}{ll} 13 & \{x^4-px^3+qx^2-rx-S=0\} \\ 15 & \{x^4-px^3+qx^2-rx+S=0\} \end{array}\right. \\ 8.\ x^3+px^2+qx+r=0 \left\{\begin{array}{ll} 14 & \{x^4+px^3+qx^2+rx-S=0\} \\ 16 & \{x^4+px+qx^2+rx+S=0\} \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2}+\frac{p^2}{8L}\backsim\frac{q}{2L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}\backsim\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH.$$

*Reg. Gen.* Describatur itaque Parabola (N A M), cujus Latus Rectum sit L (ceu 1), Axisque (a y); ad quem
1 1 ordinatim applicetur recta B A $=\frac{p}{2}$, occurrens Parabolæ in B & A: Ex puncto A (puta) ducatur Diameter, vel Axi parallela (A y); in quâ sumptâ (AD=b,
2 2 hoc est, A b $=\frac{L}{2}$, & b c $=\frac{p^2}{8L}$, deorsùm continuò
3 versus y sunt collocandæ. Tum exinde (à Puncto c,)
2 4 oportet facere c D $=\frac{q}{2L}$, eamque quidem ulteriùs deorsum versus y collocare, si in Æquatione habeatur $-q$; sursum verò, versus alteram partem, si habea-
2 tur ibi $+q$, inventumque erit Punctum D. A quo, erigatur perpendicularis ad A y, recta (DH = d, h. e.)
3 5 6 D e $=\frac{p}{4}$, & e f $=\frac{p^3}{16L^2}$, ad sinistram continuò collo-
4 7 candæ. Tum à Puncto f, oportet facere f g $=\frac{pq}{4L^2}$, eamque exinde ulteriùs ad sinistram collocare, si in
5 Æquatione habeatur $-q$; ad dextram vero si $+q$.
8 Denique ex Puncto g, oportet facere g H $=\frac{r}{2L^2}$, eamque ulteriùs exinde ad sinistram collocare, si in Æquatione p & r, iisdem signis sint adfectæ; ad dextram verò exinde, si diversis; inventumque erit Punctum H,
6 sive Circuli centrum. Quo invento, & connexâ H A, oportet ex Centro H, Circulum (N A M), describere, cujus Semidiameter sit H A, si Æquatio tantùm Cubica fuerit, hoc est, si non habeatur Quantitas S.

Ast

$$\left\{\begin{array}{l} 7.\ x^3 - px^2 + qx - r = 0 \left\{\begin{array}{l} 13\ x^4 - px^3 + qx^2 - rx - S = 0 \\ 15\ x^4 - px^3 + qx^2 - rx + S = 0 \end{array}\right. \\ 8.\ x^3 + px^2 + qx + r = 0 \left\{\begin{array}{l} 14\ x^4 + px^3 + qx^2 + rx - S = 0 \\ 16\ x^4 + px^3 + qx^2 + rx + S = 0 \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$

*Gen. Rule*

Let a Parabole (NAM) therefore be described,
whose *Latus Rectum* L (or 1), and Axe (a y); to
1 1 which, ordinately apply $BA = \frac{p}{2}$, meeting the Pa-
rabole in B and A: From the Point A (suppose),
draw the Diameter, or a Parallel to the Axe
(*viz.* A y); in which, let be taken (A D = b, *i. e.*)
2 2 3 $Ab = \frac{L}{2}$, and $bc = \frac{p^2}{8L}$, placing them both always
downwards towards y. Then from thence (from the
2 4 Point c,) make $cD = \frac{q}{2L}$, placing it indeed yet far-
ther downwards towards y, if in the Equation be had
$-q$; but upward, towards the other side, if be had
2 $+q$, and the Point D will be found. From which
Point, erect perpendicular to A y (D H = d, *i. e.*)
3 5 6 $De = \frac{p}{4}$, and $ef = \frac{p^3}{16L^2}$, both which place always
to the left hand. Then from the Point f, make
4 7 $fg = \frac{pq}{4L^2}$, placing it from thence farther to the left,
5 if in the Equation be had $-q$; but on the right, if $+q$.
8 Lastly, from the Point g, make $gH = \frac{r}{2L^2}$, placing it
thence farther to the left hand, if in the Equation p
and r are affected with the same Signs; but to the right
hand from thence, if with divers; and the Point H, or
the center of the Circle, will be found: Which found,
6 and H A connected, center H, Semidiameter H A, let
the Circle (N A M) be described, if it be only a Cubic
Equation, *i. e.* if the Quantity S be not had. But

7 Ast si habeatur S, & sit $-$ S, oportet ulteriùs in
hâc lineâ A H, productâ utrinque, ex unâ parte su-
8 9 mere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$; descri-
9 10
10 11 ptoque Semicirculo, cujus Diameter I K, erigere A L
ad A H perpendicularem, quæ occurrat huic Semicir-
culo (I L K) in puncto L.
11 Quòd si verò habeatur $+$ S; oportet insuper in
alio Semicirculo, cujus Diameter sit A H, inscribere
11 12 $AZ = AL$ inventæ.
12 Circulus igitur descriptus, transiens per L, si sit $-$ S;
per Z verò, si sit $+$ S, secare vel tangere possit Para-
bolam, in tot Punctis, quot Æquatio diversas admittet
12 Radices; è quibus si ad Diametrum (A y) demittantur
Perpendiculares, habebuntur omnes Æquationis radi-
ces, tàm falsæ, quàm veræ. Quarum quidem veræ
14 (ut N O) ad sinistram partem Diametri cadent, &
falsæ (ut M O) ad ejus dextram, si in Æquatione
15 habeatur $-$ p: Sed contra, si habeatur ibi $+$ p, veræ
quidem cadent ad dextram (ut M O), falsæ verò (ut
N O) ad sinistram.

---

$$\left\{\begin{array}{ll} 1.\ x^3 - px^2 - qx - r = 0 & \left\{\begin{array}{l} 1\ \ x^4 - px^3 - qx^2 - rx - S = 0 \\ 3\ \ x^4 - px^3 - qx^2 - rx + S = 0 \end{array}\right. \\ 2.\ x^3 + px^2 - qx + r = 0 & \left\{\begin{array}{l} 2\ \ x^4 + px^3 - qx^2 + rx - S = 0 \\ 4\ \ x^4 + px^3 - qx^2 + rx + S = 0 \end{array}\right. \end{array}\right.$$

*Demonstrat.*

$$Ab + bc + cD = b = AD.$$

2+3+4 13 $$\frac{L}{2} + \frac{p^2}{8 L} + \frac{q}{2 L} = b = AD.$$

$$De + ef + fg + gH = d = DH.$$

5+6+7+8 14 $$\frac{p}{4} + \frac{p^3}{16 L^2} + \frac{pq}{4 L^2} + \frac{r}{2 L^2} = d = DH.$$

47, e 1 $$AD^2 + DH^2 = HA^2.$$

Q.13 + Q.14. 15 $$b^2 + d^2 = (HA^2 =)\ \text{Q. Rad. in Cubic.}$$ *Fig.* 41.

AI ×

7 But if S be had, and it be $-S$, then must there be farther in this Line AH, both ways produced, taken on
8 9 the one side $AI = L$, and on the other $AK = \frac{S}{L^3}$;
9 10 and a Semicircle being described, whose Diameter IK,
10 11 must be erected AL perpendicular to AH, which may meet this Semicircle (ILK) in the Point L.

11 But if $+S$ be had, there must moreover in another Semicircle, whose Diameter is AH, be inscribed
11 12 $AZ = AL$ found.

12 A Circle therefore described, whose Center H, passing through L if it be $-S$, but through Z if $+S$, will cut or touch the Parabole in so many Points, as the Equation will admit diversity of Roots; from which, if Perpendiculars be demitted to the Diame-
12 ter (Ay), all the Roots of the Equation, as well false as true, will be had: Of which, those truly which are
14 true (as NO) will fall on the left side of the Diameter, and the false (as MO) on the right, if in the
15 Equation be had $-p$: But on the contrary, if it be $+p$, the true indeed will fall on the right hand (as MO), but the false (as NO) on the left.

| | | | | |
|---|---|---|---|---|
| 9 × 10 | 16 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2$.<br>$(L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2$. | | |
| 47, e 1<br>15 + 16 | 17 | $AH^2 + AL^2 = HL^2$.<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. | In Biquadr. si $-S$. | *Fig.* 42. |
| 47, e 1<br>15 − 16 | 18 | $AH^2 - AZ^2 = HZ^2$.<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. | In Biquadr. si $+S$. | *Fig.* 43. |
| *Supp.* | 19 | $NO = x$. | | |
| *Supp.* | 19 | $MO = -x$. | | |

NO—

| | | |
|---|---|---|
| 19 — 1 | 20 | NO — (OF, u) BA = (NF, u) OR. <br> $x - \frac{p}{2} = OR.$ |
| 19 + 1 | 20 | MO + (OF, u) BA = (MF, u) OR. <br> $-x + \frac{p}{2} = OR.$ |
| *Ob para.* | 21 | L . NO :: OR . AO. <br> $L \;.\; x :: x - \frac{p}{2} \;.\; \frac{x^2}{L} - \frac{px}{2L} = AO.$ |
| *Ob para.* | 21 | L . MO :: OR . AO. <br> $L \;.\; -x :: -x + \frac{p}{2} \;.\; \frac{x^2}{L} - \frac{px}{2L} = AO.$ |
| 21 ∽ 15 | 22 | AO ∽ AD = (DO, u) HP. <br> $\frac{x^2}{L} - \frac{px}{2L}\,(-b, u) - \frac{L}{2} - \frac{p^3}{8L} - \frac{q}{2L^2} = HP.$ |
| © | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ |
| 23 | 24 | h.e. $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ |
| 19 — 14 | 25 | NO — (OP, u) DH = PN. <br> $x\,(-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PN.$ |
| 19 + 14 | 25 | MO + (OP, u) DH = PM. <br> $-x\,(+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PM.$ |
| © | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^2x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PN^2.$ |
| © | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PM^2.$ |
| 47, e 1 <br> 24 + 26 | 27 | $HP^2 + PN^2 = HN^2.$ <br> $b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)$ Q.Rad. |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$ ; in $\frac{L^2}{x}$. |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 - qx - r = 0.$ *Q.e.d.* in Cubic. |

*Fig.* 41.

$\frac{x^4}{L}$

| | | |
|---|---|---|
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 31 | $x^4 - px^3 - qx^2 - rx = S$. |
| *Transp.* | 32 | $x^4 - px^3 - qx^2 - rx - S = 0$. in Biquadr. si $-S$. *Fig.* 42. |
| $27 = 18$ | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 - qx^2 - rx = - S$. |
| *Transp.* | 35 | $x^4 - px^3 - qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 43. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| $19 + 1$ | 20 | $\left\{\begin{aligned} & MO + (OF, u)\, BA = (MF, u)\, OR. \\ & x + \frac{p}{2} = OR. \end{aligned}\right.$ |
| $19 - 1$ | 20 | $\left\{\begin{aligned} & NO - (OF, u)\, BA = (NF, u)\, OR. \\ & -x - \frac{p}{2} = OR. \end{aligned}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{aligned} & L \,.\, MO :: OR \,.\, AO. \\ & L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{aligned}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{aligned} & L \,.\, NO :: OR \,.\, AO. \\ & L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{aligned}\right.$ |
| $21 \backsim 13$ | 22 | $\left\{\begin{aligned} & AO \backsim AD = (DO, u)\, HP. \\ & \frac{x^2}{L} + \frac{px}{2L}\, (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{aligned}\right.$ |
| $\textcircled{}$ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$. |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$. |
| $19 + 14$ | 25 | $\left\{\begin{aligned} & MO + (OP, u)\, DH = PM. \\ & x\, (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PM. \end{aligned}\right.$ |

| | | | |
|---|---|---|---|
| 19 — 14 | 25 | $\begin{cases} NO - (OP, \mu)\ DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^2}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PN. \end{cases}$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2,$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^3} + \frac{rx}{L^2} = PN^2.$ | |
| 47, e. 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HM^2 =)\ Q.Rad. \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 - qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 41. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 - qx^2 + rx = S.$ | |
| *Transp.* | 32 | $x^4 + px^3 - qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. si $-S$. | *Fig.* 42. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2};$ in $L^2$. | |
| *Transp.* | 34 | $x^4 + px^3 - qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad. si $+S$. | *Fig.* 43. |

*Illustrat.*

1. $\begin{cases} x^3 - \overset{p.}{8}x^2 - \overset{q.}{324}x - \overset{r.}{1440} = 0 \\ x^3 - 0.8x^2 - 3.24x - 1.440 = 0 \end{cases}$

$NO = x = 24.$ $\begin{cases} MO = -x = -10. \\ mo = -x = -6. \end{cases}$ *Fig.* 41.

2. $\begin{cases} x^3 + \overset{p.}{8}x^2 - \overset{q.}{324}x + \overset{r.}{1440} = 0 \\ x^3 + 0.8x^2 - 3.24x + 1.440 = 0 \end{cases}$

$\begin{cases} MO = x = 10. \\ mo = x = 6. \end{cases}$ $NO = -x = -24.$

$$\left\{\begin{array}{lll} \frac{p}{2}=0.4. & \frac{p^2}{4}=0.16. & \frac{p^3}{8}=0.064. \\ \frac{p}{4}=0.2. & \frac{p^2}{8}=0.08. & \frac{p^3}{16}=0.032. \end{array}\right.$$

***Central.*** Fig.41.

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.08 \\ +\frac{q}{2}=1,62 \\ \hline b=2.20=AD. \end{array}\right\} \begin{array}{l} +\frac{p}{4}=0.2 \\ +\frac{p^3}{16}=0.032 \\ +\frac{pq}{4}=0.648 \\ +\frac{r}{2}=0.720 \\ \hline d=1.600=DH. \end{array}$$

$$1\left\{\begin{array}{l} x^4 \overset{p.}{-} 6x^3 \overset{q.}{-} 328x^2 \overset{r.}{-} 2304x \overset{s.}{-} 4608=0 \\ x^4-0.6x^3-3.28x^2-2.304x-0.4608=0 \end{array}\right\}$$

$$NO=x=24.\left\{\begin{array}{l} MO=-x=-8. \\ mo=-x=-6. \\ mo=-x=-4. \end{array}\right.$$

$$2\left\{\begin{array}{l} x^4 \overset{p.}{+} 6x^3 \overset{q.}{-} 328x^2 \overset{r.}{+} 2304x \overset{s.}{-} 4608=0 \\ x^4+0.6x^3-3.28x^2+2.304x-0.4608=0 \end{array}\right\}$$

$$\left.\begin{array}{l} MO=x=8 \\ mo=x=6 \\ mo=x=4 \end{array}\right\} NO=-x=-24.$$ Fig.42.

$$\left\{\begin{array}{lll} \frac{p}{2}=0.3. & \frac{p^2}{4}=0.09. & \frac{p^3}{8}=0.027. \\ \frac{p}{4}=0.15. & \frac{p^2}{8}=0.045. & \frac{p^3}{16}=0.0135. \end{array}\right.$$

*Central.*

$$\left.\begin{aligned} +\frac{L}{2} &= 0.5 \\ +\frac{p^2}{8} &= 0.045 \\ +\frac{q}{2} &= 1.64 \\ \hline b &= 2.185 = AD \end{aligned}\right\} \quad \begin{aligned} +\frac{p}{4} &= 0.15 \\ +\frac{p^3}{16} &= 0.0135 \\ +\frac{pq}{4} &= 0.4920 \\ +\frac{r}{2} &= 1.152 \\ \hline d &= 1.8075 = DH. \end{aligned}$$

*Fig.* 42.

$$3\left\{\begin{aligned} &\overset{}{x^4} - \overset{p.}{6x^3} - \overset{q.}{381x^2} - \overset{r.}{326x} + 2208 = 0 \\ &x^4 - 0.6x^3 - 3.81x^2 + 0.326x + 0.2208 = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} NO &= x = 23. \\ no &= x = 2. \end{aligned}\right\}\left\{\begin{aligned} MO &= -x = -16. \\ mo &= -x = -3. \end{aligned}\right.$$

$$4\left\{\begin{aligned} &x^4 + \overset{p.}{6x^3} - \overset{q.}{381x^2} + \overset{r.}{326x} + \overset{s.}{2208} = 0 \\ &x^4 + 0.6x^3 - 3.81x^2 + 0.326x + 0.2208 = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} MO &= x = 16. \\ mo &= x = 3. \end{aligned}\right\}\left\{\begin{aligned} NO &= -x = -23. \\ no &= -x = -2. \end{aligned}\right.$$

$$\left\{\begin{aligned} \frac{p}{2} &= 0.3. & \frac{p^2}{4} &= 0.09. & \frac{p^3}{8} &= 0.027. \\ \frac{p}{4} &= 0.15. & \frac{p^2}{8} &= 0.045, & \frac{p^3}{16} &= 0.0135. \end{aligned}\right.$$

*Fig.* 43.

*Central.*

$$\left.\begin{aligned} +\frac{L}{2} &= 0.5 \\ +\frac{p^2}{8} &= 0.045 \\ +\frac{q}{2} &= 1.905 \\ \hline b &= 2.450 = AD \end{aligned}\right\} \quad \begin{aligned} +\frac{p}{4} &= 0.15 \\ +\frac{p^3}{16} &= 0.0135 \\ +\frac{pq}{4} &= 0.5715. \\ +\frac{r}{2} &= 0.163 \\ \hline d &= 0.8980 = DH. \end{aligned}$$

3. $x^3$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 - qx + r = 0 \\ 4.\ x^3 + px^2 - qx - r = 0 \end{array}\right\} \begin{array}{l} \left\{\begin{array}{l} 5 \\ 7 \end{array}\right\} \left\{\begin{array}{l} x^4 - px^3 - qx^2 + rx - S = 0 \\ x^4 - px^3 - qx^2 + rx + S = 0 \end{array}\right\} \\ \left\{\begin{array}{l} 6 \\ 8 \end{array}\right\} \left\{\begin{array}{l} x^4 + px^3 - qx^2 - rx - S = 0 \\ x^4 + px^3 - qx^2 - rx + S = 0 \end{array}\right\} \end{array}$$

*Cas.* 1. Ubi $\frac{q}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2}$

*Demonstrat.*

2+3+4 13 $\left\{\begin{array}{l} Ab + bc + cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} + \frac{q^2}{2L} = b = AD. \end{array}\right.$

5+6+7−8 14 $\left\{\begin{array}{l} De + ef + fg - gH = d = DH. \\ \frac{q}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH. \end{array}\right.$

47, e 1 / Q. 13 + Q. 14. 15 $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =)\ \text{Q. Rad. in Cubic.} \end{array}\right.$

9 × 10 16 $\left\{\begin{array}{l} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, e 1 / 15 + 16 17 $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)\ \text{Q. Rad.} \end{array}\right\}$ In Biquadr. si − S. Fig. 44. Fig. 45.

47, e 1 / 15 − 16 18 $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)\ \text{Q. Rad.} \end{array}\right\}$ In Biquadr. si + S. Fig. 46.

Supp. 19 NO = x.

Supp. 19 MO = −x.

19 − 1 20 $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$

19 + 1 20 $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$

L . NO

| | | | |
|---|---|---|---|
| | | $\left\{ L \,.\, NO :: OR \,.\, AO. \right.$ | |
| *Ob para.* | 20 | $\left\{ L : x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \right.$ | |
| | | $\left\{ L \,.\, MO :: OR \,.\, AO. \right.$ | |
| *Ob para.* | 21 | $\left\{ L : -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \right.$ | |
| | | $\left\{ AO \backsim AD = (DO, u)\ HP. \right.$ | |
| $21 \backsim 13$ | 22 | $\left\{ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \right.$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| | | $\left\{ NO - (OP, u)\ DH = PN. \right.$ | |
| $19 - 14$ | 25 | $\left\{ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \right.$ | |
| | | $\left\{ MO + (OP, u)\ DH = PM. \right.$ | |
| $19 + 14$ | 25 | $\left\{ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \right.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} + \frac{rx}{L^2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2.$ | |
| | | $\left\{ HP^2 + PN^2 = HN^2. \right.$ | |
| 47, e 1<br>$24 + 26$ | 27 | $\left\{ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{b^2} = (HN^2 =)\ Q.Rad. \right.$ | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 - qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 44. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 - qx^2 + rx = S.$ | |
| *Transp.* | 32 | $x^4 - px^3 - qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi $-$ S. | *Fig.* 45. |
| | | | $\frac{x^4}{L^2}$ |

| | | |
|---|---|---|
| $27 = 18$ | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 - qx^2 + rx = -S$. |
| *Transp.* | 35 | $x^4 - px^3 - qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 46. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| $19 + 1$ | 20 | $\begin{cases} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| $19 - 1$ | 20 | $\begin{cases} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| $21 \backsim 13$ | 22 | $\begin{cases} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{cases}$ |
| ☉ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$ |
| 23 | 24 | h.e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$. |
| $19 + 14$ | 25 | $\begin{cases} MO + (OP, u)\, DH = PM. \\ x\ (+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \end{cases}$ |
| $19 - 14$ | 25 | $\begin{cases} NO - (OP, u)\, DH = PN. \\ -x - (d, u)\, \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \end{cases}$ |
| ☉ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} - \frac{rx}{L^2} = PM^2$. |
| ☉ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} - \frac{rx}{L^2} = PN^2$. |

HP²

47, e 1
24 + 26 | 27 | $\left\{ \begin{array}{l} HP^2 + PM^2 \pm HM^2 \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =) \text{ Q. Rad.} \end{array} \right.$

27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$.

$\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 - qx - r = 0$. *Q. e. d.* in Cubic. *Fig. 44.*

27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$.

$\times L^2$ | 31 | $x^4 + px^3 - qx^2 - rx = S$.

*Transp.* | 32 | $x^4 + px^3 - qx^2 - rx - S = 0$. *Q. e. d.* in Biquad. $-S$. *Fig. 45.*

25 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$.

$\times L^2$ | 34 | $x^4 + px^3 - qx^2 - rx = -S$.

*Transp.* | 35 | $x^4 + px^3 - qx^2 - rx + S = 0$. *Q. e. d.* in Biquad. $+S$. *Fig. 46.*

*Illustrat.*

$p. \quad q. \quad r.$

3 $\left\{ \begin{array}{l} x^3 - 21x^2 - 110x + 2662 = 0 \\ x^3 - 2.1x^2 - 1.10x + 2.662 = 0 \end{array} \right\}$

$\left. \begin{array}{l} NO = x = 20. \text{ } \textit{proximè.} \\ no = x = 12.24 \text{ } \textit{proximè.} \end{array} \right\}$ $MO = -x = -11.$

$p. \quad q. \quad r.$

4 $\left\{ \begin{array}{l} x^3 + 21x^2 - 110x - 2662 = 0 \\ x^3 + 2.1x^2 - 1.10x - 2.662 = 0 \end{array} \right\}$

$MO = x = 11.$ $\left\{ \begin{array}{l} NO = -x = -20. \text{ } \textit{proximè.} \\ no = -x = -12.24 \text{ } \textit{proximè.} \end{array} \right.$ *Fig. 44.*

$\left\{ \begin{array}{l} \frac{p}{2} = 1.05. \quad \frac{p^2}{4} = 1.1025. \quad \frac{p^3}{8} = 1.157625. \\ \frac{p}{4} = 0.525. \quad \frac{p^2}{8} = 0.55125. \quad \frac{p^3}{16} = 0.5788125. \end{array} \right.$

*Central.*

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.55125 \\ +\frac{q}{2}=0.55 \\ \hline b=1.60125=AD \end{array}\right\} \quad \begin{array}{l} +\frac{p}{4}=0.525 \\ +\frac{p^3}{16}=0.5788125 \\ +\frac{pq}{4}=0.57550 \\ \hline 1.6793125 \\ \hline -\frac{r}{2}=1.3310 \\ \hline d=0.3483125=DH. \end{array}$$

Fig. 44.

$$5\left\{\begin{array}{l} \overset{}{x^4}-\overset{p.}{18}x^3-\overset{q.}{205\tfrac{1}{4}}x^2+\overset{r.}{3033}x-\overset{s.}{5117}=0 \\ x^4-1.8x^3-2.05\tfrac{1}{4}x^2+3.033x-0.5117=0 \end{array}\right\}$$

$$\left.\begin{array}{l} NO=x=21.5 \\ no=x=8.5 \\ no=x=2. \end{array}\right\} \quad MO=-x=-14.$$

$$6\left\{\begin{array}{l} x^4+\overset{p.}{18}x^3-\overset{q.}{205\tfrac{1}{4}}x^2-\overset{r.}{3033}x-\overset{s.}{5117}=0 \\ x^4+1.8x^3-2.05\tfrac{1}{4}x^2-3.033x-0.5117=0 \end{array}\right\}$$

$$MO=x=14.\left\{\begin{array}{l} NO=-x=-21.5 \\ no=-x=-8.5 \\ no=-x=-2. \end{array}\right.$$

Fig. 45.

$$\left\{\begin{array}{lll} \frac{p}{2}=0.9. & \frac{p^2}{4}=0.81. & \frac{p^3}{8}=0.729. \\ \frac{p}{4}=0.45. & \frac{p^2}{8}=0.405. & \frac{p^3}{16}=0.3645. \end{array}\right.$$

*Central.*

$$\left.\begin{aligned} +\frac{L}{2} &= 0.5 \\ +\frac{p^2}{8} &= 0.405 \\ +\frac{q}{2} &= 1.02625 \\ \hline b &= 1.93125 = AD \end{aligned}\right\} \qquad \begin{aligned} +\frac{p}{4} &= 0.45 \\ +\frac{p^3}{16} &= 0.3645 \\ +\frac{pq}{4} &= 0.923625 \\ \hline & \phantom{=}\ 1.738125 \\ \hline -\frac{r}{2} &= 1.5165 \\ \hline d &= 0.221625 = DH. \end{aligned}$$

*Fig.* 45.

$$7 \left\{\begin{array}{l} \qquad\quad\ \ p. \qquad\qquad q. \qquad\qquad\quad r. \qquad\qquad s. \\ x^4 - 12x^3 - 195x^2 + 550x + 3000 = 0 \\ x^4 - 1.2x^3 - 1.95x^2 + 0.550x + 0.3000 = 0 \end{array}\right\}$$

$$\left.\begin{aligned} NO = x &= 20. \\ no = x &= \ \ 5. \end{aligned}\right\}\left\{\begin{aligned} MO &= -x = -10. \\ mo &= -x = -\ \ 3. \end{aligned}\right.$$

$$8 \left\{\begin{array}{l} \qquad\quad\ \ p. \qquad\qquad q. \qquad\qquad\quad r. \qquad\qquad s. \\ x^4 + 12x^3 - 195x^2 - 550x + 3000 = 0 \\ x^4 + 1.2x^3 - 1.95x^2 - 0.550x + 0.3000 = 0 \end{array}\right\}$$

$$\left.\begin{aligned} MO = x &= 10. \\ mo = x &= \ \ 3. \end{aligned}\right\}\left\{\begin{aligned} NO &= -x = -20. \\ no &= -x = -\ \ 5. \end{aligned}\right.$$

$$\left\{\begin{array}{lll} \dfrac{p}{2} = 0.6. & \dfrac{p^2}{4} = 0.36. & \dfrac{p^3}{8} = 0.216. \\ \dfrac{p}{4} = 0.3. & \dfrac{p^2}{8} = 0.18. & \dfrac{p^3}{16} = 0.108. \end{array}\right.$$

*Fig.* 46.

*Central.*

$$\left.\begin{aligned} +\frac{L}{2} &= 0.5 \\ +\frac{p^2}{8} &= 0.18 \\ +\frac{q}{2} &= 0.975 \\ \hline b &= 1.655 = AD \end{aligned}\right\} \qquad \begin{aligned} +\frac{p}{4} &= 0.3 \\ +\frac{p^3}{16} &= 0.108 \\ +\frac{pq}{4} &= 0.585 \\ \hline & \phantom{=}\ 0.993 \\ \hline -\frac{r}{2} &= 0.275 \\ \hline d &= 0.718 = DH. \end{aligned}$$

*Cas.*

*Cas.* 2. Ubi $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2}$.

*Demonstrat.*

| | | | |
|---|---|---|---|
| 2+3+4 | 13 | $Ab + bc + cD = b = AD.$<br>$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD.$ | |
| 8−7−6−5 | 14 | $Hg - gf - fe - eD = d = DH.$<br>$\frac{r}{2L^2} - \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH.$ | |
| 47, *e* 1<br>*Q.* 13.+<br>*Q.* 14. | 15 | $AD^2 + DH^2 = HA^2.$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. | *Fig.* 47. |
| 9 × 10 | 16 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2.$<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | |
| 47, *e* 1<br>15 + 16 | 17 | $AH^2 + AL^2 = HL^2.$<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. } In Biquadr. si − S. | *Fig.* 48. |
| 47, *e* 1<br>15 — 16 | 18 | $HA^2 - AZ^2 = HZ^2.$<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. } In Biquadr. si + S. | *Fig.* 49. |
| *Supp.* | 19 | $NO = x.$ | |
| *Supp.* | 19 | $MO = -x.$ | |
| 19 — 1 | 20 | $NO -$ (OF, u) $BA =$ (NF, u) $OR.$<br>$x - \frac{p}{2} = OR.$ | |
| 19 + 1 | 20 | $MO +$ (OF, u) $BA =$ (MF, u) $OR.$<br>$-x + \frac{p}{2} = OR.$ | |

| | | | |
|---|---|---|---|
| | | $L \,.\, NO :: OR \,.\, AO.$ | |
| *Ob para.* | 21 | $L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.$ | |
| | | $L \,.\, MO :: OR \,.\, AO.$ | |
| *Ob para.* | 21 | $L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.$ | |
| | | $AO \backsim AD = (DO, u)\; HP.$ | |
| $21 \backsim 13$ | 22 | $\frac{x^2}{L} - \frac{px}{2L}\; (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP.$ | |
| © | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| | | $NO + (OP, u)\; DH = PN.$ | |
| $19 + 14$ | 25 | $x\; (+d, u) + \frac{r}{2L^2} - \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.$ | |
| | | $MO - (OP, u)\; DH = PM.$ | |
| $19 - 14$ | 25 | $-x\; (-d, u) - \frac{r}{2L^2} + \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM.$ | |
| © | 26 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| © | 26 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| 47, e 1 | | $HP^2 + PN^2 = HN^2.$ | |
| $24 + 26$ | 27 | $b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)$ Q.Rad. | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 - qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.*47. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 - qx^2 + rx = S.$ | |
| *Transp.* | 32 | $x^4 - px^3 - qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. si $-S$. | *Fig.*48. |

$\frac{x^4}{L^2}$

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 - qx^2 + rx = -S$. |
| *Transp.* | 35 | $x^4 - px^3 - qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 49. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| 19 + 1 | 20 | $\begin{cases} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| 19 − 1 | 20 | $\begin{cases} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| 21 ∽ 13 | 22 | $\begin{cases} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{cases}$ |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$. |
| 19 − 14 | 25 | $\begin{cases} MO - (OP, u)\, DH = PM. \\ x\ (-d, u) - \frac{r}{2L^2} + \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ |
| 19 + 14 | 25 | $\begin{cases} NO + (OP, u)\, DH = PN. \\ -x\ (+d, u) + \frac{r}{2L^2} - \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ |
| ⊙ | 26 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2$. |
| ⊙ | 26 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2$. |

$HP^2$

| | | | |
|---|---|---|---|
| 47, e 1<br>24 + 26 | 27 | $\left\{ \begin{array}{l} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =) \text{ Q.Rad.} \end{array} \right.$ | |
| 27 = 15<br>$\times \frac{L^2}{x}$ | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| | 29 | $x^3 + px^2 - qx - r = 0$. *Q. e. d.* in Cubic. | *Fig*. 47. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 - qx^2 - rx = S$. | |
| *Transp.* | 32 | $x^4 + px^3 - qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. ſi $-S$. | *Fig*. 48. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 - qx^2 - rx = -S$. | |
| *Transp.* | 35 | $x^4 + px^3 - qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig*. 49. |

*Illuſtrat.*

$$3 \left\{ \begin{array}{l} \quad\quad\;\; p. \quad\quad\quad q. \quad\quad\quad r. \\ x^3 - 8x^2 - 240x + 2304 = 0 \\ x^3 - 0.8x^2 - 2.40x + 2.304 = 0 \end{array} \right\}$$

$$\left. \begin{array}{l} NO = x = 12. \\ no = x = 12: \end{array} \right\} \quad MO = -x = -16.$$

$$4 \left\{ \begin{array}{l} \quad\quad\;\; p. \quad\quad\quad q. \quad\quad\quad r. \\ x^3 + 8x^2 - 240x - 2304 = 0 \\ x^3 + 0.8x^2 - 2.40x - 2.304 = 0 \end{array} \right\}$$

$$MO = x = 16. \quad \left\{ \begin{array}{l} NO = -x = -12. \\ no = -x = -12. \end{array} \right.$$ *Fig*. 47.

$$\left\{ \begin{array}{lll} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 1.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{array} \right.$$

Central.

*Central.*

$$\left.\begin{aligned} +\frac{L}{2} &= 0.5 \\ +\frac{p^2}{8} &= 0.08 \\ +\frac{q}{2} &= 1.20 \\ \hline b &= 1.78 = AD. \end{aligned}\right\} \qquad \begin{aligned} +\frac{r}{2} &= 1.152 \\ \hline -\frac{p}{4} &= 0.2 \\ -\frac{p^3}{16} &= 0.032 \\ -\frac{pq}{4} &= 0.480 \\ \hline & \phantom{=}\ 0.712 \\ \hline d &= 0.440 = DH. \end{aligned}$$

Fig. 47.

$$\begin{array}{l} \qquad\quad p. \qquad\qquad q. \qquad\qquad r. \qquad\qquad s. \\ 5 \left\{\begin{array}{l} x^4 - 8\,x^3 - 208\,x^2 + 2432\,x - 6144 = 0 \\ x^4 - 0.8\,x^3 - 2.08\,x^2 + 2.432\,x - 0.6144 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO = x = 12. \\ no = x = 8. \\ no = x = 4. \end{array}\right\} \quad MO = -x = -16.$$

$$\begin{array}{l} \qquad\quad p. \qquad\qquad q. \qquad\qquad r. \qquad\qquad s. \\ 6 \left\{\begin{array}{l} x^4 + 8\,x^3 - 208\,x^2 - 2432\,x - 6144 = 0 \\ x^4 + 0.8\,x^3 - 2.08\,x^2 - 2.432\,x - 0.6144 = 0 \end{array}\right\} \end{array}$$

$$MO = x = 16. \quad \left\{\begin{array}{l} NO = -x = -12. \\ no = -x = -8. \\ no = -x = -4. \end{array}\right.$$

Fig. 48.

$$\left\{\begin{array}{lll} \dfrac{p}{2} = 0.4. & \dfrac{p^2}{4} = 0.16. & \dfrac{p^3}{8} = 0.064. \\ \dfrac{p}{4} = 0.2. & \dfrac{p^2}{8} = 0.08. & \dfrac{p^3}{16} = 0.032. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.08 \\ +\frac{q}{2}=1.04 \\ \hline b=1.62=AD. \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=1.216 \\ \hline -\frac{p}{4}=0.2 \\ -\frac{p^3}{16}=0.032 \\ -\frac{pq}{4}=0.416 \\ \hline \quad 0.648 \\ \hline d=0.568=DH. \end{array}$$

Fig. 48.

$$\begin{array}{lllll} & p. & q. & r. & s. \end{array}$$

$$7 \left\{\begin{array}{l} x^4 - 6x^3 - 333x^2 + 2322x + 9720 = 0 \\ x^4 - 0.6x^3 - 3.33x^2 + 2.322x + 0.9720 = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} NO = x = 15. \\ no = x = 12. \end{array}\right\}\left\{\begin{array}{l} MO = -x = -18. \\ mo = -x = -3. \end{array}\right.$$

$$\begin{array}{lllll} & p. & q. & r. & s. \end{array}$$

$$8 \left\{\begin{array}{l} x^4 + 6x^3 - 333x^2 - 2322x + 9720 = 0 \\ x^4 + 0.6x^3 - 3.33x^2 - 2.322x + 0.9720 = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} MO = x = 18. \\ mo = x = 3. \end{array}\right\}\left\{\begin{array}{l} NO = -x = -15. \\ no = -x = -12. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.3. & \frac{p^2}{4} = 0.09. & \frac{p^3}{8} = 0.027. \\ \frac{p}{4} = 0.15. & \frac{p^2}{8} = 0.045. & \frac{p^3}{16} = 0.0135. \end{array}\right.$$

Fig. 49.

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.045 \\ +\frac{q}{2}=1.665 \\ \hline b=2.210=AD. \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=1.161 \\ \hline -\frac{p}{4}=0.15. \\ -\frac{p^3}{16}=0.0135 \\ -\frac{pq}{4}=0.4995 \\ \hline \quad 0.6630 \\ \hline d=0.4980=DH. \end{array}$$

5. $x^3$

$$\left\{\begin{array}{l} 5.\ x^3 - px^2 + qx + r = 0 \left\{\begin{array}{ll} 9 & x^4 - px^3 + qx^2 + rx - S = 0 \\ 11 & x^4 - px^3 + qx^2 + rx + S = 0 \end{array}\right. \\ 6.\ x^3 + px^2 + qx - r = 0 \left\{\begin{array}{ll} 10 & x^4 + px^3 + qx^2 - rx - S = 0 \\ 12 & x^4 + px^3 + qx^2 - rx + S = 0 \end{array}\right. \end{array}\right.$$

*Cas.* 1. Ubi $\frac{L}{2} + \frac{p^2}{8L} \sqsupset \frac{q}{2L}$; & $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{pq}{4L^2} + \frac{r}{2L^2}$.

## *Demonstrat.*

2+3−4 | 13 | $\left\{\begin{array}{l} Ab + bc - cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD. \end{array}\right.$

5+6−7−8 | 14 | $\left\{\begin{array}{l} De + ef - fg - gH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH. \end{array}\right.$

47, e 1, Q. 13 + Q. 14. | 15 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \end{array}\right.$ Q. Rad. in Cubic. — *Fig.* 50.

9 × 10 | 16 | $\left\{\begin{array}{l} AI \times AK = \end{array}\right.$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2$.
$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2$.

47, e 1, 15 + 16 | 17 | $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \end{array}\right.$ Q. Rad. } In Biquadr. si − S. — *Fig.* 51.

47, e 1, 15 − 16 | 18 | $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \end{array}\right.$ Q. Rad. } In Biquadr. si + S. — *Fig.* 52.

*Supp.* | 19 | $NO = x$.

*Supp.* | 19 | $MO = -x$.

19 − 1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$

19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$

*Ob para.* 21 $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$

*Ob para.* 21 $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$

21 ∽ 13 22 $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$

⊙ 23 $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$

23 24 he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$

19 − 14 25 $\left\{\begin{array}{l} NO - (OP, u)\, DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \end{array}\right.$

19 + 14 25 $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \end{array}\right.$

⊙ 26 $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PN^2.$

⊙ 26 $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2.$

47, e 1
24 + 26 27 $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)\ Q.Rad. \end{array}\right.$

27 = 15 28 $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$.

$\times \frac{L^2}{x}$ 29 $x^3 - px^2 + qx + r = 0$. *Q.e.d.* in Cubic. *Fig.* 50.

27 = 17 30 $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$.

$\times L^2$ 31 $x^4 - px^3 + qx^2 + rx = S.$

*Transp.* 32 $x^4 - px^3 + qx^2 + rx - S = 0$. *Q.e.d.* in Biquad. si $-S$. *Fig.* 51.

$\frac{x^4}{L^2}$

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 + rx = -S$. |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 52. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| 19 + 1 | 20 | $MO + (OF, u)\ BA = (MF, u)\ OR$. <br> $x + \frac{p}{2} = OR$. |
| 19 − 1 | 20 | $NO - (OF, u)\ BA = (NF, u)\ OR$. <br> $-x - \frac{p}{2} = OR$. |
| *Ob para.* | 21 | $L\ .\ MO :: OR\ .\ AO$. <br> $L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO$. |
| *Ob para.* | 21 | $L\ .\ NO :: OR\ .\ AO$. <br> $L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO$. |
| 21 ∽ 13 | 22 | $AO \backsim AD = (DO, u)\ HP$. <br> $\frac{x^2}{L} + \frac{px}{2L}\ (\backsim b, u) \backsim \frac{L}{2} \backsim \frac{p^2}{8L} + \frac{q}{2L} = HP$. |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$. |
| 19 + 14 | 25 | $MO + (OP, u)\ DH = PM$. <br> $x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PM$. |
| 19 − 14 | 25 | $NO - (OP, u)\ DH = PN$. <br> $-x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PN$. |
| ⊙ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PM^2$. |
| ⊙ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PN^2$. |

| | | | |
|---|---|---|---|
| 47, e 1; 24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)\ \text{Q.Rad.} \end{cases}$ | |
| 27 = 15; $\times \frac{L^2}{x}$ | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| | 29 | $x^3 + px^2 + qx - r = 0$. *Q. e. d.* in Cubic. | *Fig.* 50. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 + qx^2 - rx = S$. | |
| *Transp.* | 32 | $x^4 + px^3 + qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. si $-S$. | *Fig.* 51. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 + qx^2 - rx = -S$. | |
| *Transp.* | 35 | $x^4 + px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.* 52. |

*Illustrat.*

$$5\ \begin{cases} x^3 - \overset{p.}{24}x^2 + \overset{q.}{20}x + \overset{r.}{1200} = 0 \\ x^3 - 2.4x^2 + 0.20x + 1.200 = 0 \end{cases}$$

$$\left.\begin{matrix} NO = x = 20. \\ no = x = 10. \end{matrix}\right\}\quad MO = -x = -6.$$

$$6\ \begin{cases} x^3 + \overset{p.}{24}x^2 + \overset{q.}{20}x - \overset{r.}{1200} = 0 \\ x^3 + 2.4x^2 + 0.20x - 1.200 = 0 \end{cases}$$

$$MO = x = 6.\quad \begin{cases} NO = -x = -20. \\ no = -x = -10. \end{cases}$$ *Fig.* 50.

$$\begin{cases} \frac{p}{2} = 1.2. & \frac{p^2}{4} = 1.44. & \frac{p^3}{8} = 1.728. \\ \frac{p}{4} = 0.6. & \frac{p^2}{8} = 0.72. & \frac{p^3}{16} = 0.864. \end{cases}$$

*Central.*

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2} = 0.5 \\ +\frac{p^2}{8} = 0.72 \\ \hline + \quad 1.22 \\ -\frac{q}{2} = 0.10 \\ \hline b = 1.12 = AD. \end{array}\right\} \quad \begin{array}{l} +\frac{p}{4} = 0.6 \\ +\frac{p^3}{16} = 0.864 \\ \hline \quad 1.464 \\ \hline -\frac{pq}{4} = 0.120 \\ -\frac{r}{2} = 0.600 \\ \hline \quad 0.720 \\ \hline d = 0.744 = DH. \end{array}$$

*Fig.* 50.

$$\begin{array}{l} \quad\quad p. \quad\quad\quad q. \quad\quad\quad r. \quad\quad\quad s. \\ 9\left\{\begin{array}{l} x^4 - 30x^3 + 128x^2 + 2016x - 11520 = 0 \\ x^4 - 3.0x^3 + 1.28x^2 + 2.016x - 1.1520 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO = x = 20. \\ no = x = 12. \\ no = x = 6. \end{array}\right\} \quad MO = -x = -8.$$

$$\begin{array}{l} \quad\quad p. \quad\quad\quad q. \quad\quad\quad r. \quad\quad\quad s. \\ 10\left\{\begin{array}{l} x^4 + 30x^3 + 128x^2 - 2016x - 11520 = 0 \\ x^4 + 3.0x^3 + 1.28x^2 - 2.016x - 1.1520 = 0 \end{array}\right\} \end{array}$$

*Fig.* 51.

$$MO = x = 8. \quad \left\{\begin{array}{l} NO = -x = -20. \\ no = -x = -12. \\ no = -x = -6. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.5. & \frac{p^2}{4} = 2.25. & \frac{p^3}{8} = 3.375. \\ \frac{p}{4} = 0.75. & \frac{p^2}{8} = 1.125. & \frac{p^3}{16} = 1.6875. \end{array}\right.$$

Cen-

Central.

$$+\frac{L}{2}=0.5 \qquad +\frac{p}{4}=0.75$$
$$+\frac{p^2}{8}=1.125 \qquad +\frac{p^3}{16}=1.6875$$
$$+\;1.625 \qquad +\;2.4375$$
$$\frac{q}{2}=0.64 \qquad -\frac{pq}{4}=0.9600$$
$$b=0.985=AD. \qquad -\frac{r}{2}=1.008$$
$$1.9680$$
$$d=0.4695=DH.$$

Fig. 51.

$$\begin{array}{cccccc} & p. & q. & r. & s. & \\ \text{II}\left\{\begin{array}{l} x^4 \\ x^4 \end{array}\right. & \begin{array}{l} -\,26x^3 \\ -\,2.6x^3 \end{array} & \begin{array}{l} +\,20x^2 \\ +\,0.20x^2 \end{array} & \begin{array}{l} +\,1832x \\ -\,1.832x \end{array} & \begin{array}{l} +\,3360=0 \\ +\,0.3360=0 \end{array} & \right\} \end{array}$$

$$\begin{array}{l} NO=x=20. \\ no=x=14. \end{array}\Big\}\Big\{\begin{array}{l} MO=-x=-6. \\ mo=-x=-2. \end{array}$$

$$\begin{array}{cccccc} & p. & q. & r. & s. & \\ \left\{\begin{array}{l} x^4 \\ x^4 \end{array}\right. & \begin{array}{l} +\,26x^3 \\ +\,2.6x^3 \end{array} & \begin{array}{l} +\,20x^2 \\ +\,0.20x^2 \end{array} & \begin{array}{l} -\,1832x \\ -\,1.832x \end{array} & \begin{array}{l} +\,3360=0 \\ +\,0.3360=0 \end{array} & \right\} \end{array}$$

$$\begin{array}{l} MO=x=6. \\ mo=x=2. \end{array}\Big\}\Big\{\begin{array}{l} NO=-x=-20. \\ no=-x=-14. \end{array}$$

$$\left\{\begin{array}{lll} \frac{p}{2}=1.3. & \frac{p^2}{4}=1.69. & \frac{p^3}{8}=2.197. \\ \frac{p}{4}=0.65. & \frac{p^2}{8}=0.845. & \frac{p^3}{16}=1.0985. \end{array}\right.$$

Fig. 52.

Central.

$$+\frac{L}{2}=0.5 \qquad +\frac{p}{4}=0.65.$$
$$+\frac{p^2}{8}=0.845 \qquad +\frac{p^3}{16}=1.0985$$
$$1.345 \qquad 1.7485$$
$$-\frac{q}{2}=0.10 \qquad -\frac{pq}{4}=0.1300$$
$$b=1.245=AD. \qquad -\frac{r}{2}=0.916$$
$$1.0460$$
$$d=0.7025=DH.$$

*Cas.* 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$; & $\frac{pq}{4L^2} + \frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$.

*Demonstrat.*

| | | | | |
|---|---|---|---|---|
| 4—3—2 | 13 | $\begin{cases} Dc - cb - bA = b = AD. \\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD. \end{cases}$ | | |
| 7+8−6−5 | 14 | $\begin{cases} Hg + gf - fe - eD = d = DH. \\ \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{cases}$ | | |
| 47, *e* 1 *Q*.13 + *Q*.14. | 15 | $\begin{cases} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{cases}$ | | *Fig*. 53. |
| 9 × 10 | 16 | $\begin{cases} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | | |
| 47, *e* 1 15 + 16 | 17 | $\begin{cases} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{ Q.Rad.} \end{cases}$ | In Biquadr. si − S. | *Fig*. 54. |
| 47, *e* 1 15 − 16 | 18 | $\begin{cases} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{ Q.Rad.} \end{cases}$ | In Biquadr. si + S. | *Fig*. 55. |
| *Supp*. | 19 | $NO = x.$ | | |
| *Supp*. | 19 | $MO = -x.$ | | |
| 1 — 19 | 20 | $\begin{cases} (OF, u)\ BA - NO = (NF, u)\ OR. \\ \frac{p}{2} - x = OR. \end{cases}$ | | |
| 19 + 1 | 20 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | | |

L . NO

| | | |
|---|---|---|
| | | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: \frac{p}{2} - x \,.\, \frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| 13 − 21 | 22 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} - \frac{px}{2L} + \frac{x^2}{L} = HP. \end{array}\right.$ |
| 13 + 21 | 22 | $\left\{\begin{array}{l} AD + AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP. \end{array}\right.$ |
| $\odot$ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ |
| 23 | 24 | he, $b^4 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ |
| 19 + 14 | 25 | $\left\{\begin{array}{l} NO + (OP, u)\ DH = PN. \\ x\,(+ d, u) + \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ |
| 14 − 19 | 25 | $\left\{\begin{array}{l} (OP, u)\ DH - MO = PM. \\ (d, u)\ \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} + x = PM. \end{array}\right.$ |
| $\odot$ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ |
| $\odot$ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ |
| 47, e 1<br>24 + 26 | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)\ Q.Rad. \end{array}\right.$ |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx + r = 0$. *Q.e.d.* in Cubic. *Fig.* 53. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 + rx = S.$ |
| *Transp.* | 32 | $x^4 - px^3 + qx^2 + rx - S = 0$. *Q.e.d.* in Biquad. si − S. *Fig.* 54. |

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 + rx = -S.$ |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad. si $+S$. *Fig.* 55. |
| *Supp.* | 19 | $MO = x.$ |
| *Supp.* | 19 | $NO = -x.$ |
| 19 + 1 | 20 | $MO + (OF, u)\ BA = (MF, u)\ OR.$<br>$x + \frac{p}{2} = OR.$ |
| 1 − 19 | 20 | $(OF, u)\ BA - NO = (NF, u)\ OR.$<br>$\frac{p}{2} + x = OR.$ |
| *Ob para.* | 21 | $L\ .\ MO :: OR\ .\ AO.$<br>$L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO.$ |
| *Ob para.* | 21 | $L\ .\ NO :: OR\ .\ AO.$<br>$L\ .\ -x :: \frac{p}{2} + x\ .\ -\frac{px}{2L} - \frac{x^2}{L} = AO.$ |
| 13 + 21 | 22 | $AD + AO = (DO, u)\ HP.$<br>$(b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} + \frac{px}{2L} = HP.$ |
| 13 − 21 | 22 | $AD - AO = (DO, u)\ HP.$<br>$(b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{px}{2L} + \frac{x^2}{L} = HP.$ |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} + \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} - \frac{p^3x}{8L^2} - x^2 - \frac{px}{2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - x^2 - \frac{px}{2} = HP^2.$ |
| 14 − 19 | 25 | $(OP, u)\ DH - MO = PM.$<br>$(d, u)\ \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} - x = PM.$ |
| 19 + 14 | 25 | $NO + (OP, u)\ DH = PN.$<br>$-x\ (+d, u) + \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.$ |

| | | | |
|---|---|---|---|
| ⊙ | 26 | $d^2 + x^2 - \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $\left\{ \begin{array}{l} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)\ Q.Rad. \end{array} \right.$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx - r = 0$. *Q.e.d.* in Cubic. | *Fig.* 53. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$. | 31 | $x^4 + px^3 + qx^2 - rx = S.$ | |
| *Transp.* | 32 | $x^4 + px^3 + qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. si $-S$. | *Fig.* 54. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$. | 34 | $x^4 + px^3 + qx^2 - rx = -S.$ | |
| *Transp.* | 35 | $x^4 + px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.* 55. |

*Illustrat.*

5 $\left\{ \begin{array}{l} \quad\quad\ \ p. \quad\quad\quad\ q. \quad\quad\quad\ r. \\ x^3 - 12x^2 + 200x + 4200 = 0 \\ x^3 - 1.2x^2 + 2.00x + 4.200 = 0 \end{array} \right\}$

$MO = -x = -10.$

6 $\left\{ \begin{array}{l} \quad\quad\ \ p. \quad\quad\quad\ q. \quad\quad\quad\ r. \\ x^3 + 12x^2 + 200x - 4200 = 0 \\ x^3 + 1.2x^2 + 2.00x - 4.200 = 0 \end{array} \right\}$ *Fig.* 53.

$MO = x = 10.$

$\left\{ \begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array} \right.$

Central.

*Central.*

$$+\frac{q}{2L} = 1.00 \qquad +\frac{pq}{4} = 0.600$$

$$-\frac{p^2}{8} = 0.18 \qquad +\frac{r}{2} = 2.100$$

$$-\frac{L}{2} = 0.5 \qquad 2.700$$

$$0.68 \qquad -\frac{p^3}{16} = 0.108$$

$$b = 0.32 = AD. \qquad -\frac{p}{4} = 0.3$$

$$0.408$$

$$d = 2.292 = DH.$$

Fig. 53.

$$9\left\{\begin{array}{l} x^4 - \overset{p.}{12}\,x^3 + \overset{q.}{216}\,x^2 + \overset{r.}{916}\,x - \overset{s.}{6168} = 0 \\ x^4 - 1.2\,x^3 + 2.16\,x^2 + 0.916\,x - 0.6168 = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} NO = x = 3.8 + \\ MO = -x = -6. \end{array}\right.$$

$$10\left\{\begin{array}{l} x^4 + \overset{p.}{12}\,x^3 + \overset{q.}{216}\,x^2 - \overset{r.}{916}\,x - \overset{s.}{6168} = 0 \\ x^4 + 1.2\,x^3 + 2.16\,x^2 - 0.916\,x - 0.6168 = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} MO = 6 = x. \\ NO = -x = -3.8 + \end{array}\right.$$

Fig. 54.

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array}\right.$$

*Central.*

$$+\frac{q}{2L}=1.08 \quad -\frac{p^2}{8}=0.18 \quad -\frac{L}{2}=0.5 \quad \overline{0.68} \quad b=0.40=AD.$$

$$+\frac{pq}{4}=0.648 \quad +\frac{r}{2}=0.458 \quad \overline{1.106} \quad -\frac{p^3}{16}=0.108 \quad -\frac{p}{4}=0.3 \quad \overline{0.408} \quad d=0.698=DH.$$

Fig. 54.

$$11\left\{\begin{array}{l} x^4 \overset{p.}{-12x^3} \overset{q.}{+200x^2} \overset{r.}{+2344x} \overset{s.}{+2976}=0 \\ x^4-1.2x^3+2.00x^2+2.344x+0.2976=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=-x=-6. \\ mo=-x=-1.47+ \end{array}\right.$$

$$12\left\{\begin{array}{l} x^4 \overset{p.}{+12x^3} \overset{q.}{+200x^2} \overset{r.}{-2344x} \overset{s.}{+2976}=0 \\ x^4+1.2x^3+2.00x^2-2.344x+0.2976=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=x=6. \\ mo=x=1.47+ \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=0.6. & \frac{p^2}{4}=0.36. & \frac{p^3}{8}=0.216. \\ \frac{p}{4}=0.3. & \frac{p^2}{8}=0.18. & \frac{p^3}{16}=0.108. \end{array}\right.$$

Fig. 55.

*Central.*

$$+\frac{q}{2}=1.00 \quad -\frac{p^2}{8}=0.18 \quad -\frac{L}{2}=0.5 \quad \overline{0.68} \quad b=0.32=AD.$$

$$+\frac{pq}{4}=0.600 \quad +\frac{r}{2}=1.172 \quad \overline{1.772} \quad -\frac{p^3}{16}=0.108 \quad -\frac{p}{4}=0.3. \quad \overline{0.408} \quad d=1.364=DH.$$

*Cas.* 3. Ubi $\frac{L}{2}+\frac{p^2}{8L}\sqsupset\frac{q}{2L}$ ; & $\frac{pq}{4L^2}+\frac{r}{2L^2}\sqsupset\frac{p}{4}+\frac{p^3}{16L^2}$.

*Demonstrat.*

| | | | | |
|---|---|---|---|---|
| 2+3−4 | 13 | $\begin{cases} Ab+bc-cD=b=AD. \\ \frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=b=AD. \end{cases}$ | | |
| 8+7−6−5 | 14 | $\begin{cases} Hg+gf-fe-eD=d=DH. \\ \frac{r}{2L^2}+\frac{pq}{4L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=d=DH. \end{cases}$ | | |
| 47, *e* 1<br>*Q*. 13. + *Q*. 14. | 15 | $\begin{cases} AD^2+DH^2=HA^2. \\ b^2+d^2=(HA^2=)\ \text{Q. Rad. in Cubic.} \end{cases}$ | | *Fig.* 56. |
| 9 × 10 | 16 | $\begin{cases} AI\times AK=(\text{ob Circl.})\ AL^2=(\text{per constr.})\ AZ^2. \\ (L\times\frac{S}{L^3}=)\ \frac{S}{L^2}=AL^2=AZ^2. \end{cases}$ | | |
| 47, *e* 1<br>15 + 16 | 17 | $\begin{cases} HA^2+AL^2=HL^2. \\ b^2+d^2+\frac{S}{L^2}=(HL^2=)\text{Q.Rad.} \end{cases}$ | In Biquadr. si − S. | *Fig.* 57. |
| 15 − 16 | 18 | $\begin{cases} HA^2-AZ^2=HZ^2. \\ b^2+d^2-\frac{S}{L^2}=(HZ^2=)\text{Q.Rad.} \end{cases}$ | In Biquadr. si + S. | *Fig.* 58. |
| *Supp.* | 19 | $NO=x.$ | | |
| *Supp.* | 19 | $MO=-x.$ | | |
| 1 − 19 | 20 | $\begin{cases} (OF,u)\ BA-NO=(NF,u)\ OR. \\ \frac{p}{2}-x=OR. \end{cases}$ | | |
| 19 + 1 | 20 | $\begin{cases} MO+(OF,u)\ BA=(MF,u)\ OR. \\ -x+\frac{p}{2}=OR. \end{cases}$ | | |

L . NO

| | | | |
|---|---|---|---|
| | | $(L \,.\, NO :: OR \,.\, AO.$ | |
| *Ob para.* | 21 | $L \,.\, x :: \frac{p}{2} - x \,.\, \frac{px}{2} - \frac{x^2}{L} = AO.$ | |
| | | $L \,.\, MO :: OR \,.\, AO.$ | |
| *Ob para.* | 21 | $L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.$ | |
| | | $AO + AD = (DO,\ u)\ HP.$ | |
| $21 + 13$ | 22 | $\frac{px}{2L} - \frac{x^2}{L}\ (+b,\ u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP.$ | *Fig.* 57. |
| | | $AO - AD = (DO,\ u)\ HP.$ | |
| $21 - 13$ | 22 | $\frac{x^2}{L} - \frac{px}{2L}\ (-b,\ u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP.$ | *Fig.* 56, 57, 58 |
| $\odot$ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - x^2 - \frac{p^2x^2}{4L^2} = HP^2.$ | |
| 23 | 24 | $he,\ b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - x^2 = HP^2.$ | |
| | | $NO + (OP,\ u)\ DH = PN.$ | |
| $19 + 14$ | 25 | $x\ (+d,\ u)\ \frac{r}{2L^2} + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.$ | |
| | | $MO - (OP,\ u)\ DH = PM.$ | |
| $19 - 14$ | 25 | $-x\ (-d,\ u) - \frac{r}{2L^2} - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM.$ | |
| $\odot$ | 26 | $d^2 + x^2 + \frac{rx}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| $\odot$ | 26 | $d^2 + x^2 + \frac{rx}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| | | $HP^2 + PN^2 = HN^2.$ | |
| 47, e 1<br>$24 + 26$ | 27 | $b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)$ Q. Rad. | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 56. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 + rx = S.$ | |
| *Transp.* | 32 | $x^4 - px^3 + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. si $-S$. | *Fig.* 57. |

33

| | | | |
|---|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 + rx = -S$. | |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.* 58. |
| *Supp.* | 19 | $MO = x$. | |
| *Supp.* | 19 | $NO = -x$. | |
| 19 + 1 | 20 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ | |
| 1 − 19 | 20 | $\begin{cases} (OF, u)\ BA - NO = (NF, u)\ OR. \\ \frac{p}{2} + x = OR. \end{cases}$ | |
| *Ob para.* | 21 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 21 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: \frac{p}{2} + x\ .\ -\frac{px}{2L} - \frac{x^2}{L} = AO. \end{cases}$ | |
| 21 − 13 | 22 | $\begin{cases} AO - AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{cases}$ | *Fig.* 56, 57, 58 |
| 21 + 13 | 22 | $\begin{cases} AO + AD = (DO, u)\ HP. \\ -\frac{px}{2L} - \frac{x^2}{L}\ (+b, u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{cases}$ | *Fig.* 57. |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} = HP^2$ | |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$. | |
| 19 − 14 | 25 | $\begin{cases} MO - (OP, u)\ DH = PM. \\ x\ (-d, u) - \frac{r}{2L^2} - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ | |
| 19 + 14 | 25 | $\begin{cases} NO + (OP, u)\ DH = PN. \\ -x\ (+d, u) + \frac{r}{2L^2} + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ | |

$d^2 +$

| | | | |
|---|---|---|---|
| ☉. | 26 | $d^2 + x^2 - \frac{rx}{L^2} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ | |
| ☉ | 26 | $d^2 + x^2 - \frac{rx}{L^2} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =) \text{ Q.Rad.} \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx - r = 0$. *Q.e.d.* in Cubic. | *Fig.* 56. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 + qx^2 - rx = + S.$ | |
| *Transp.* | 32 | $x^4 + px^3 + qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. si $-S$. | *Fig.* 57. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 + qx^2 - rx = - S.$ | |
| *Transp.* | 35 | $x^4 + px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.* 58. |

*Illustrat.*

$$5 \quad \begin{array}{lllll} & p. & q. & r. & \\ x^3 & - 16x^2 & + 100x & + 1072 & = 0 \\ x^3 & - 1.6x^2 & + 1.00x & + 1.072 & = 0 \end{array}$$

$MO = - x = - 6.$

$$6 \quad \begin{array}{lllll} & p. & q. & r. & \\ x^3 & + 16x^2 & + 100x & - 1072 & = 0 \\ x^3 & + 1.6x^2 & + 1.00x & - 1.072 & = 0 \end{array}$$ *Fig.* 56.

$MO = x = 6.$

$$\begin{cases} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{cases}$$

Central.

*Central.*

$$+\frac{L}{2} = 0.5 \qquad\qquad +\frac{r}{2L^2} = 0.536$$

$$+\frac{p^2}{8} = 0.32 \qquad\qquad +\frac{pq}{4L^2} = 0.400$$

$$0.82 \qquad\qquad 0.936$$ Fig. 56.

$$-\frac{q}{2} = 0.50 \qquad\qquad -\frac{p^3}{16} = 0.256$$

$$b = 0.32 = AD. \qquad\qquad -\frac{p}{4} = 0.4$$

$$0.656$$

$$d = 0.280 = DH.$$

$$9 \begin{cases} \overset{}{x^4} - \overset{p.}{12x^3} + \overset{q.}{80x^2} + \overset{r.}{500x} - \overset{s.}{3768} = 0 \\ x^4 - 1.2x^3 + 0.80x^2 + 0.500x - 0.3768 = 0 \end{cases}$$

$$\begin{cases} NO = x = 5.16 - \\ MO = -x = -6. \end{cases}$$

$$10 \begin{cases} x^4 + \overset{p.}{12x^3} + \overset{q.}{80x^2} - \overset{r.}{500x} - \overset{s.}{3768} = 0 \\ x^4 + 1.2x^3 + 0.80x^2 - 0.500x - 0.3768 = 0 \end{cases}$$ Fig. 57.

$$\begin{cases} MO = x = 6. \\ NO = -x = -5.16 - \end{cases}$$

$$\begin{cases} \frac{p}{2} = 0.6. \quad \frac{p^2}{4} = 0.36. \quad \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. \quad \frac{p^2}{8} = 0.18. \quad \frac{p^3}{16} = 0.108. \end{cases}$$

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.18 \\ \hline \quad 0.68 \\ -\frac{q}{2}=0.40 \\ \hline b=0.28=AD. \end{array}\right\} \begin{array}{l} +\frac{r}{2}=0.250 \\ +\frac{pq}{4}=0.240 \\ \hline \quad 0.490 \\ -\frac{p^3}{16}=0.108 \\ -\frac{p}{4}=0.3 \\ \hline \quad 0.408 \\ d=0.082=DH. \end{array}$$

*Fig.* 57.

$$11\left\{\begin{array}{l} x^4 \overset{p.}{-16x^3} \overset{q.}{+40x^2} \overset{r.}{+1365x} \overset{s.}{+1998}=0 \\ x^4-1.6x^3+0.40x^2+1.365x+0.1998=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=-x=-6. \\ mo=-x=-1.6- \end{array}\right.$$

$$12\left\{\begin{array}{l} x^4 \overset{p.}{+16x^3} \overset{q.}{+40x^2} \overset{r.}{-1365x} \overset{s.}{+1998}=0 \\ x^4+1.6x^3+0.40x^2-1.365x+0.1998=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=x=6. \\ mo=x=1.6- \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=0.8. & \frac{p^2}{4}=0.64. & \frac{p^3}{8}=0.512. \\ \frac{p}{4}=0.4. & \frac{p^2}{8}=0.32. & \frac{p^3}{16}=0.256. \end{array}\right.$$

*Fig.* 58.

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.32 \\ \hline \quad 0.82 \\ -\frac{q}{2}=0.20 \\ \hline b=0.62=AD. \end{array}\right\} \begin{array}{l} +\frac{r}{2}=0.6825 \\ +\frac{pq}{4}=0.160 \\ \hline \quad 0.8425 \\ -\frac{p^3}{16}=0.256 \\ -\frac{p}{4}=0.4. \\ \hline \quad 0.656 \\ d=0.1865=DH. \end{array}$$

$$\left\{\begin{array}{l} 7.\ x^3 - px^2 + qx - r = o \left\{\begin{array}{l} 13\ \{ x^4 - px^3 + qx^2 - rx - S = o \\ 15\ \{ x^4 - px^3 + qx^2 - rx + S = o \end{array}\right. \\ 8.\ x^3 + px^2 + qx + r = o \left\{\begin{array}{l} 14\ \{ x^4 + px^3 + qx^2 + rx - S = o \\ 16\ \{ x^4 + px^3 + qx^2 + rx + S = o \end{array}\right. \end{array}\right.$$

*Caf.* 1. Ubi $\frac{L}{2} + \frac{p^2}{8L} \sqsupset \frac{q}{2L}$ ; & $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} \sqsupset \frac{pq}{4L^2}$.

*Demonſtrat.*

2+3—4 | 13 | $\left\{\begin{array}{l} Ab + bc - cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD. \end{array}\right.$

5+6+8—7 | 14 | $\left\{\begin{array}{l} De + ef + fg - gH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = d = DH. \end{array}\right.$

47, e 1 Q. 13. + Q. 14. | 15 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. $\end{array}\right.$ *Fig.* 59.

9 × 10 | 16 | $\left\{\begin{array}{l} AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2. \\ (L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, e 1 15 + 16 | 17 | $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q. Rad. $\end{array}\right\}$ In Biquadr. ſi — S. *Fig.* 60.

47, e 1 15 — 16 | 18 | $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q. Rad. $\end{array}\right\}$ In Biquadr. ſi + S. *Fig.* 61.

*Supp.* | 19 | NO $= x$.

*Supp.* | 19 | MO $= -x$.

19 — 1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$

19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 21 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| 21 ∓ 13 | 22 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{cases}$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 19 − 14 | 25 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} + \frac{pq}{4L^2} = PN. \end{cases}$ | |
| 19 + 14 | 25 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = PM. \end{cases}$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} + \frac{pqx}{2L^2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} + \frac{pqx}{2L^2} = PM^2.$ | |
| 47, *e* 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)\ Q{:}Rad. \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx - r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 59. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 - rx = S.$ | |
| *Transp.* | 32 | $x^4 - px^3 + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquad. si $-S$. | *Fig.* 60. |

$\frac{x^4}{L^2}$

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = -S$. |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 61. |
| *Supp.* | 19 | $no = x$. |
| 1 − 19 | 20 | $(OF, u)\ BA - no = (nF, u)\ OR$.<br>$\frac{p}{2} - x = OR$. |
| *Ob para.* | 21 | $L\ .\ no :: OR\ .\ AO$.<br>$L\ .\ x :: \frac{p}{2} - x\ .\ \frac{px}{2L} - \frac{x^2}{L} = AO$. |
| 21 + 13 | 22 | $AO + AD = (DO, u)\ HP$.<br>$\frac{px}{2L} - \frac{x^2}{L}\ (+b, u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP$. |
| ⊖ | 23 | &c. Vide pag. 124, &c. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| 19 + 1 | 20 | $MO + (OF, u)\ BA = (MF, u)\ OR$.<br>$x + \frac{p}{2} = OR$. |
| 19 − 1 | 20 | $NO - (OF, u)\ BA = (NF, u)\ OR$.<br>$-x - \frac{p}{2} = OR$. |
| *Ob para.* | 21 | $L\ .\ MO :: OR\ .\ AO$.<br>$L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO$. |
| *Ob para.* | 21 | $L\ .\ NO :: NO\ .\ AO$.<br>$L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO$. |
| 21 ∽ 13 | 22 | $AO \backsim AD = (DO, u)\ HP$.<br>$\frac{x^2}{L} + \frac{px}{2L}\ (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP$. |

$b^2 +$

| | | | |
|---|---|---|---|
| ⊙ | 23 | $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-x^2-\frac{p^2x^2}{4L^2}-\frac{px}{2}-\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2$ | |
| 23 | 24 | h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2-\frac{px}{2}-\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2$ | |
| $19+14$ | 25 | $\begin{cases} MO+(OP,u)\,DH=PM. \\ x\,(+d,u)+\frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}-\frac{pq}{4L^2}=PM. \end{cases}$ | |
| $19-14$ | 25 | $\begin{cases} NO-(OP,u)\,DH=PN. \\ -x\,(-d,u)-\frac{p}{4}-\frac{p^3}{16L^2}-\frac{r}{2L^2}+\frac{pq}{4L^2}=PN. \end{cases}$ | |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{rx}{L^2}-\frac{pqx}{2L^2}=PM^2.$ | |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{rx}{L^2}-\frac{pqx}{2L^2}=PN^2.$ | |
| 47, *e* 1<br>$24+26$ | 27 | $\begin{cases} HP^2+PM^2=HM^2. \\ b^2+d^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=(HM^2=)\ \text{Q.Rad.} \end{cases}$ | |
| $27=15$ | 28 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3+px^2+qx+r=0.$ *Q. e. d.* in Cubic. | *Fig.* 59. |
| $27=17$ | 30 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4+px^3+qx^2+rx=S.$ | |
| *Transp.* | 32 | $x^4+px^3+qx^2+rx-S=0.$ *Q. e. d.* in Biquad. si $-S$. | *Fig.* 60. |
| $27=18$ | 33 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4+px^3+qx^2+rx=-S.$ | |
| *Transp.* | 35 | $x^4+px^3+qx^2+rx+S=0.$ *Q. e. d.* in Biquad. si $+S$. | *Fig.* 61. |
| *Supp.* | 19 | $no=-x.$ | |
| $1-19$ | 20 | $\begin{cases} (OF,u)\,BA-no=(nF,u)\,OR. \\ \frac{p}{2}+x=OR. \end{cases}$ | |

L . no

*Ob para.* 21 $\begin{cases} L \,.\, no :: OR \,.\, AO. \\ L \,.\, -x :: \frac{p}{2}+x \,.\, -\frac{px}{2L}-\frac{x^2}{L}=AO. \end{cases}$

21 + 13 22 $\begin{cases} AO+AD=(DO,\, u)\ HP. \\ -\frac{px}{2L}-\frac{x^2}{L}\ (+b,\, u)+\frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=HP. \end{cases}$

@ 23 &c. Vide pag. 125, &c.

*Illustrat.*

$$7\ \begin{cases} \overset{p.}{} \\ x^3-40x^2+432x-1152=0 \\ x^3-4.0x^2+4.32x-1.152=0 \end{cases}$$

(p. q. r. over the coefficients 40, 432, 1152)

$$\begin{cases} NO=x=24. \\ no=x=12. \\ no=x=\ \ 4. \end{cases}$$

$$8\ \begin{cases} x^4+40x^2+432x+1152=0 \\ x^4+4.0x^2+4.32x+1.152=0 \end{cases}$$ *Fig.* 59.

(p. q. r. over the coefficients 40, 432, 1152)

$$\begin{cases} NO=-x=-24. \\ no=-x=-12. \\ no=-x=-\ \ 4. \end{cases}$$

$$\begin{cases} \frac{p}{2}=2.0. & \frac{p^2}{4}=4.0. & \frac{p^3}{8}=8.0. \\ \frac{p}{4}=1.0. & \frac{p^2}{8}=2.0. & \frac{p^3}{16}=4.0. \end{cases}$$

Central.

**Central.**

$$+\frac{L}{2}=0.5 \qquad +\frac{p}{4}=1.0$$
$$+\frac{p^2}{8}=2.0 \qquad +\frac{p^3}{16}=4.0$$
$$2.5 \qquad +\frac{r}{2}=0.576$$
$$-\frac{q}{2}=2.16 \qquad 5.576$$
$$b=0.34=AD. \qquad -\frac{pq}{4}=4.32$$
$$d=1.256=DH.$$

Fig. 59.

$$13\begin{cases} x^4 - \overset{p.}{34}x^3 + \overset{q.}{344}x^2 - \overset{r.}{704}x - \overset{s.}{3072}=0 \\ x^4 - 3.4x^3 + 3.44x^2 - 0.794x - 0.3072=0 \end{cases}$$

$$\left.\begin{array}{l} NO=x=16. \\ no=x=12. \\ no=x=8. \end{array}\right\} MO=-x=-2.$$

$$14\begin{cases} x^4 + \overset{p.}{34}x^3 + \overset{q.}{344}x^2 + \overset{r.}{704}x - \overset{s.}{3072}=0 \\ x^4 + 3.4x^3 + 3.44x^2 + 0.704x - 0.3072=0 \end{cases}$$

Fig. 60.

$$MO=x=2. \begin{cases} NO=-x=-16. \\ no=-x=-12. \\ no=-x=-8. \end{cases}$$

$$\begin{cases} \frac{p}{2}=1.7. \quad \frac{p^2}{4}=2.89. \quad \frac{p^3}{8}=4.913. \\ \frac{p}{4}=0.85. \quad \frac{p^2}{8}=1.445. \quad \frac{p^3}{16}=2.4565. \end{cases}$$

Cen-

*Central.*

$$\left.\begin{aligned} &+\frac{L}{2}=0.5\\ &+\frac{p^2}{8}=1.445\\ \hline &\quad 1.945\\ &-\frac{q}{2}=1.72\\ \hline &b=0.225=AD. \end{aligned}\right\}\quad \begin{aligned} &+\frac{p}{4}=0.85\\ &+\frac{p^3}{16}=2.4565\\ &+\frac{r}{2}=0.352\\ \hline &\quad 3.6585\\ &-\frac{pq}{4}=2.9240\\ \hline &d=0.7345=DH. \end{aligned}$$

Fig. 60.

$$\begin{array}{l} \quad\; p. \qquad\quad q. \qquad\quad r. \qquad\quad s.\\ 15\left\{\begin{aligned} &x^4-40x^3+400x^2-514x+4128=0\\ &x^4-4.0x^3+4.00x^2-0.514x+0.4128=0 \end{aligned}\right\} \end{array}$$

$$\left\{\begin{aligned} &NO=23.8+\\ &no=16. \end{aligned}\right.$$

$$\begin{array}{l} \quad\; p. \qquad\quad q. \qquad\quad r. \qquad\quad s.\\ 16\left\{\begin{aligned} &x^4+40x^3+400x^2+514x+4128=0\\ &x^4+4.0x^3+4.00x^2+0.514x+0.4128=0 \end{aligned}\right\} \end{array}$$

$$\left\{\begin{aligned} &NO=-x=-23.8+\\ &no=-x=-16. \end{aligned}\right.$$

$$\left\{\begin{aligned} &\frac{p}{2}=2.0. \qquad \frac{p^2}{4}=4.0. \qquad \frac{p^3}{8}=8.0.\\ &\frac{p}{4}=1.0. \qquad \frac{p^2}{8}=2.0. \qquad \frac{p^3}{16}=4.0. \end{aligned}\right.$$

Fig. 61.

*Central.*

$$\left.\begin{aligned} &+\frac{L}{2}=0.5\\ &+\frac{p^2}{8}=2.0\\ \hline &\quad 2.5\\ &-\frac{q}{2}=2.00\\ \hline &b=0.50=AD. \end{aligned}\right\}\quad \begin{aligned} &+\frac{p}{4}=1.0\\ &+\frac{p^3}{16}=4.0\\ &+\frac{r}{2}=0.257\\ \hline &\quad 5.257\\ &-\frac{pq}{4}=4.000\\ \hline &d=1.257=DH. \end{aligned}$$

*Cas.* 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$; & $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} \sqsupset \frac{pq}{4L^2}$.

*Demonstrat.*

| | | |
|---|---|---|
| 4—3—2 | 13 | $\left\{\begin{array}{l} Dc - cb - bA = b = AD. \\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD. \end{array}\right.$ |
| 5+6+8—7 | 14 | $\left\{\begin{array}{l} De + ef + fg - gH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = d = DH. \end{array}\right.$ |
| 47, e 1 Q.13.+ Q.14. | 15 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. — $x$. $\end{array}\right.$ *Fig*. 62. |
| 9 × 10 | 16 | $\left\{\begin{array}{l} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per constr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$ |
| 47, e 1 15+16 | 17 | $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. $\end{array}\right\}$ In Biquadr. si — S. *Fig*. 63. |
| 47, e 1 15—16 | 18 | $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. $\end{array}\right\}$ In Biquadr. si + S. *Fig*. 64. |
| Supp. | 19 | $NO = x.$ |
| Supp. | 19 | $MO = -x.$ |
| 19—1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ |
| 19+1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ |

L . NO

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 21 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| 21 + 13 | 22 | $\begin{cases} AO + AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (+b, u) + \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = HP. \end{cases}$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{p^2x^2}{4L^2} - x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = HP^2.$ | |
| 19 − 14 | 25 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} + \frac{pq}{4L^2} = PN. \end{cases}$ | |
| 19 + 14 | 25 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ -x + (d, u)\ \ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = PM. \end{cases}$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} + \frac{pqx}{2L^2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} + \frac{pqx}{2L^2} = PM^2.$ | |
| 47, *e* 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)\ \text{Q.Rad.} \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx - r = 0.$ *Q. e. d.* in Cubic. | *Fig.*62. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 - rx = S.$ | |
| *Transp.* | 32 | $x^4 - px^3 + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquad. si $-S$. | *Fig.*63. |

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4-px^3+qx^2-rx=-S$. |
| *Transsp.* | 35 | $x^4-px^3+qx^2-rx+S=0$. *Q.e.d.* in Biquad. si $+S$. *Fig.* 64. |
| *Supp.* | 19 | $no=x$. |
| 1 — 19 | 20 | $(OF,u)\ BA-no=(nF,u)\ OR$.<br>$\frac{p}{2}-x=OR$. |
| *Ob para.* | 21 | $L\ .\ no :: OR\ .\ AO$.<br>$L\ .\ x :: \frac{p}{2}-x\ .\ \frac{px}{2L}-\frac{x^2}{L}=AO$. |
| 21 — 13 | 22 | $AO-AD=(DO,u)\ HP$.<br>$\frac{px}{2L}-\frac{x^2}{L}\ (-b,u)\ -\frac{q}{2L}+\frac{p^2}{8L}+\frac{L}{2}=HP$. |
| ⊕ | 23 | &c. Ut in pag. 131, &c. |
| *Supp.* | 19 | $MO=x$. |
| *Supp.* | 19 | $NO=-x$. |
| 19 + 1 | 20 | $MO+(OF,u)\ BA=(MF,u)\ OR$.<br>$x+\frac{p}{2}=OR$. |
| 19 — 1 | 20 | $NO-(OF,u)\ BA=(NF,u)\ OR$.<br>$-x-\frac{p}{2}=OR$. |
| *Ob para.* | 21 | $L\ .\ MO :: OR\ .\ AO$.<br>$L\ .\ x :: x+\frac{p}{2}\ .\ \frac{x^2}{L}+\frac{px}{2L}=AO$. |
| *Ob para.* | 21 | $L\ .\ NO :: OR\ .\ AO$.<br>$L\ .\ -x :: -x-\frac{p}{2}\ .\ \frac{x^2}{L}+\frac{px}{2L}=AO$. |
| 21 + 13 | 22 | $AO+AD=(DO,u)\ HP$.<br>$\frac{x^2}{L}+\frac{px}{2L}\ (+b,u)\ +\frac{q}{2L}-\frac{p^2}{8L}-\frac{L}{2}=HP$. |

$b^2+$

| | | | |
|---|---|---|---|
| ☉ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{p^2x^2}{4L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2$ | |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2$. | |
| $19 + 14$ | 25 | $\left\{ \begin{array}{l} MO + (OP, u)\ DH = PM. \\ x\,(+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = PM. \end{array} \right.$ | |
| $19 - 14$ | 25 | $\left\{ \begin{array}{l} NO - (OP, u)\ DH = PN. \\ -x\,(-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} + \frac{pq}{4L^2} = PN. \end{array} \right.$ | |
| ☉ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PM^2$. | |
| ☉ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PN^2$. | |
| 47, e 1<br>$24 + 26$ | 27 | $\left\{ \begin{array}{l} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HM^2 =)\ Q.Rad. \end{array} \right.$ | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx + r = 0$. *Q. e. d.* in Cubic. | *Fig.* 62. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 + qx^2 + rx = S$. | |
| *Transp.* | 32 | $x^4 + px^3 + qx^2 + rx - S = 0$. *Q. e. d.* in Biquad. si $-S$. | *Fig.* 63. |
| $27 = 18$ | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 + qx^2 + rx = -S$. | |
| *Transp.* | 35 | $x^4 + px^3 + qx^2 + rx + S = 0$. *Q. e. d.* in Biquad. si $+S$. | *Fig.* 64. |

*Illustrat.*

| | | |
|---|---|---|
| $27 = 18$ | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = - S$. |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+$ S. *Fig.* 64. |
| *Supp.* | 19 | $no = x$. |
| $1 - 19$ | 20 | $\begin{cases} (OF, u)\ BA - no = (nF, u)\ OR. \\ \frac{p}{2} - x = OR. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L\ .\ no :: OR\ .\ AO. \\ L\ .\ x :: \frac{p}{2} - x\ .\ \frac{px}{2L} - \frac{x^2}{L} = AO. \end{cases}$ |
| $21 - 13$ | 22 | $\begin{cases} AO - AD = (DO, u)\ HP. \\ \frac{px}{2L} - \frac{x^2}{L}\ (-b, u) - \frac{q}{2L} + \frac{p^2}{8L} + \frac{L}{2} = HP. \end{cases}$ |
| ⊖ | 23 | &c. Ut in pag. 131, &c. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = - x$. |
| $19 + 1$ | 20 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| $19 - 1$ | 20 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ - x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 21 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ - x :: - x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| $21 + 13$ | 22 | $\begin{cases} AO + AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (+b, u) + \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = HP. \end{cases}$ |

$b^2 +$

| | | | |
|---|---|---|---|
| ⊙ | 23 | $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-\frac{p^2x^2}{4L^2}-x^2+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=HP^2$ | |
| 23 | 24 | h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=HP^2$. | |
| $19+14$ | 25 | $\begin{cases} MO+(OP,u)\,DH=PM. \\ x(+d,u)+\frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}-\frac{pq}{4L^2}=PM. \end{cases}$ | |
| $19-14$ | 25 | $\begin{cases} NO-(OP,u)\,DH=PN. \\ -x(-d,u)-\frac{p}{4}-\frac{p^3}{16L^2}-\frac{r}{2L^2}+\frac{pq}{4L^2}=PN. \end{cases}$ | |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{rx}{L^2}-\frac{pqx}{2L^2}=PM^2$. | |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{rx}{L^2}-\frac{pqx}{2L^2}=PN^2$. | |
| 47, *e* 1<br>$24+26$ | 27 | $\begin{cases} HP^2+PM^2=HM^2. \\ b^2+d^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=(HM^2=)\ \text{Q.Rad.} \end{cases}$ | |
| $27=15$ | 28 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3+px^2+qx+r=0$. *Q. e. d.* in Cubic. | *Fig.62.* |
| $27=17$ | 30 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4+px^3+qx^2+rx=S$. | |
| *Transp.* | 32 | $x^4+px^3+qx^2+rx-S=0$. *Q.e.d.* in Biquad. si $-S$. | *Fig.63.* |
| $27=18$ | 33 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4+px^3+qx^2+rx=-S$. | |
| *Transp.* | 35 | $x^4+px^3+qx^2+rx+S=0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.64.* |

*Illustrat.*

*Illustrat.*

$$7\begin{cases} \overset{p.}{x^3 - 20x^2} + \overset{q.}{300x} - \overset{r.}{3776} = 0 \\ x^3 - 2.0x^2 + 3.00x - 3.776 = 0 \end{cases}$$

$NO = x = 16.$

$$8\begin{cases} \overset{p.}{x^3 + 20x^2} + \overset{q.}{300x} + \overset{r.}{3776} = 0 \\ x^3 + 2.0x^2 + 3.00x + 3.776 = 0 \end{cases}$$

$NO = -x = -16.$

$$\begin{cases} \frac{p}{2} = 1. \quad \frac{p^2}{4} = 1. \quad \frac{p^3}{8} = 1. \\ \frac{p}{4} = 0.5. \quad \frac{p^2}{8} = 0.5. \quad \frac{p^3}{16} = 0.5. \end{cases}$$ *Fig.62.*

*Central.*

$+\frac{q}{2} = 1.50$

$-\frac{p^2}{8} = 0.5$

$-\frac{L}{2} = 0.5$

$1.0$

$b = 0.5 = AD.$

$+\frac{p}{4} = 0.5$

$+\frac{p^3}{16} = 0.5$

$+\frac{r}{2} = 1.888$

$2.888$

$-\frac{pq}{4} = 1.5$

$d = 1.388 = DH.$

$$13\begin{cases} \overset{p.}{x^4 - 50x^3} + \overset{q.}{875x^2} - \overset{r.}{6450x} - \overset{s.}{22456} = 0 \\ x^4 - 5.0x^3 + 8.75x^2 - 6.450x - 2.2456 = 0 \end{cases}$$

$$\begin{cases} NO = x = 28. \\ MO = -x = -2.5- \end{cases}$$ *Fig.63.*

$$14\begin{cases} \overset{p.}{x^4 + 50x^3} + \overset{q.}{875x^2} + \overset{r.}{6450x} - \overset{s.}{22456} = 0 \\ x^4 + 5.0x^3 + 8.75x^2 + 6.450x - 2.2456 = 0 \end{cases}$$

$$\begin{cases} MO = x = 2.5- \\ NO = -x = -28. \end{cases}$$

$\frac{P}{2}$

$$\begin{cases} \frac{p}{2} = 2.5. & \frac{p^2}{4} = 6.25. & \frac{p^3}{8} = 15.625. \\ \frac{p}{4} = 1.25. & \frac{p^2}{8} = 3.125. & \frac{p^3}{16} = 7.8125. \end{cases}$$

Fig. 63.

*Central.*

$$+\frac{q}{2} = 4.375 \qquad +\frac{p}{4} = 1.25$$

$$-\frac{p^2}{8} = 3.125 \qquad +\frac{p^3}{16} = 7.8125$$

62. $$-\frac{L}{2} = 0.5 \qquad +\frac{r}{2} = 3.225$$

$$3.625 \qquad 12.2875$$

$$b = 0.750 = AD. \qquad -\frac{pq}{4} = 10.9375$$

$$d = 1.3500 = DH.$$

$$15 \begin{cases} \overset{p.}{x^4 - 50\,x^3} + \overset{q.}{875\,x^2} - \overset{r.}{8000\,x} + \overset{s.}{29044} = 0 \\ x^4 - 5.0\,x^3 + 8.75\,x^2 - 8.000\,x + 2.9044 = 0 \end{cases}$$

$$\begin{cases} NO = x = 27.1 \text{ ferè.} \\ no = x = 7. + \end{cases}$$

Fig. 64.

$$16 \begin{cases} \overset{p.}{x^4 + 50\,x^3} + \overset{q.}{875\,x^2} + \overset{r.}{8000\,x} + \overset{s.}{29044} = 0 \\ x^4 + 5.0\,x^3 + 8.75\,x^2 + 8.000\,x + 2.9044 = 0 \end{cases}$$

63. $$\begin{cases} NO = -x = -27.1 \text{ ferè.} \\ no = -x = -7. + \end{cases}$$

$$\begin{cases} \frac{p}{2} = 2.5. & \frac{p^2}{4} = 6.25. & \frac{p^3}{8} = 15.625. \\ \frac{p}{4} = 1.25. & \frac{p^2}{8} = 3.125. & \frac{p^3}{16} = 7.8125. \end{cases}$$

Cen-

*Central.*

| | |
|---|---|
| $+\frac{q}{2} = 4.375$ | $+\frac{p}{4} = 1.25$ |
| $-\frac{p^2}{8} = 3.125$ | $+\frac{p^3}{16} = 7.8125$ |
| $-\frac{L}{2} = 0.5$ | $+\frac{r}{2} = 4.0000$ |
| $3.625$ | $13.0625$ |
| $b = 0.750 = AD$ | $-\frac{pq}{4} = 10.9375$ |
| | $d = 2.1250 = DH.$ |

*Fig.* 64.

*Cas.* 3. Ubi $\frac{q}{2L^2} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$; & $\frac{pq}{4L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$.

*Demonstrat.*

| | | | | |
|---|---|---|---|---|
| 4—3—2 | 13 | $Dc - cb - bA = b = AD.$<br>$\frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD.$ | | |
| 7-8-6-5 | 14 | $Hg - gf - fe - eD = d = DH.$<br>$\frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH.$ | | |
| 47, *e* 1<br>*Q*. 13. + *Q*. 14. | 15 | $AD^2 + DH^2 = HA^2.$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. | | *Fig.* 65. |
| 9 × 10. | 16 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per constr.) $AZ^2$.<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | | |
| 47, *e* 1<br>15 + 16 | 17 | $HA^2 + AL^2 = HL^2.$<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. | In Biquadr.<br>si $-$ S. | *Fig.* 66. |
| 47, *e* 1<br>15 — 16 | 18 | $HA^2 - AZ^2 = HZ^2.$<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. | In Biquadr.<br>si + S. | *Fig.* 67. |

NO

| | | | |
|---|---|---|---|
| *Supp.* | 19 | $NO = x.$ | |
| *Supp.* | 19 | $MO = -x.$ | |
| 1 − 19 | 20 | $\left\{\begin{array}{l}(OF,\ u)\ BA - NO = (NF,\ u)\ OR.\\ \frac{p}{2} - x = OR.\end{array}\right.$ | |
| 19 + 1 | 20 | $\left\{\begin{array}{l}MO + (OF,\ u)\ BA = (MF,\ u)\ OR.\\ -x + \frac{p}{2} = OR.\end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l}L\ .\ NO :: OR\ .\ AO.\\ L\ .\ x :: \frac{p}{2} - x\ .\ \frac{px}{2L} - \frac{x^2}{L} = AO.\end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l}L\ .\ MO :: OR\ .\ AO.\\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO.\end{array}\right.$ | |
| 13 − 21 | 22 | $\left\{\begin{array}{l}AD - AO = (DO,\ u)\ HP.\\ (b,\ u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP.\end{array}\right.$ | *Fig.* {65, 66, 67 |
| 13 + 21 | 22 | $\left\{\begin{array}{l}AD + AO = (DO,\ u)\ HP.\\ (b,\ u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP.\end{array}\right.$ | *Fig.* 67. |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 19 + 14 | 25 | $\left\{\begin{array}{l}NO + (PO,\ u)\ DH = PN.\\ x\ (+d,\ u) + \frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.\end{array}\right.$ | |
| 14 − 19 | 25 | $\left\{\begin{array}{l}(OP,\ u)\ DH - MO = PM.\\ (d,)\ \frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} + x = PM.\end{array}\right.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |

| | | | |
|---|---|---|---|
| 47, e 1<br>24 + 26 | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)\ \text{Q.Rad.} \end{array}\right.$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx - r = 0$. *Q. e. d.* in Cubic. | *Fig.* 65. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 - rx = S$. | |
| *Transp.* | 32 | $x^4 - px^3 + qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. si $-S$. | *Fig.* 66. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = -S$. | |
| *Transp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. si $+S$. | *Fig.* 67. |
| *Supp.* | 19 | $MO = x$. | |
| *Supp.* | 19 | $NO = -x$. | |
| 19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ | |
| 1 − 19 | 20 | $\left\{\begin{array}{l} (OF, u)\ BA - NO = (NF, u)\ OR. \\ \frac{p}{2} + x = OR. \end{array}\right.$ | |
| *Ob. para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob. para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: \frac{p}{2} + x\ .\ - \frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ | |
| 13 + 21 | 22 | $\left\{\begin{array}{l} AD + AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} + \frac{px}{2L} = HP. \end{array}\right.$ | *Fig.* 67. |
| 13 − 21 | 22 | $\left\{\begin{array}{l} AD - AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{px}{2L} + \frac{x^2}{L} = HP. \end{array}\right.$ | *Fig.* $\left\{\begin{array}{l} 65 \\ 66 \\ 67 \end{array}\right.$ |

$b^2 +$

| | | | |
|---|---|---|---|
| ⊙ | 23 | $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{p^2x^2}{4L^2}-\frac{px}{2}-x^2=HP^2$ | |
| 23 | 24 | h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}-x^2=HP^2.$ | |
| 14 — 19 | 25 | $\begin{cases}(OP,\ u)\ DH-MO=PM.\\(d,\ u)\ \frac{pq}{4L^2}-\frac{r}{2L^2}-\frac{p^3}{16L^2}-\frac{p}{4}-x=PM.\end{cases}$ | |
| 19 + 14 | 25 | $\begin{cases}NO+(OP,\ u)\ DH=PN.\\-x(+d,\ u)+\frac{pq}{4L^2}-\frac{r}{2L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=PN.\end{cases}$ | |
| ⊙ | 26 | $d^2+x^2-\frac{pqx}{2L^2}+\frac{rx}{L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PM^2.$ | |
| ⊙ | 26 | $d^2+x^2-\frac{pqx}{2L^2}+\frac{rx}{L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PN^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $\begin{cases}HP^2+PM^2=HM^2.\\b^2+d^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=(HM^2=)\ Q.Rad.\end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3+px^2+qx+r=0.$ *Q. e. d.* in Cubic. | *Fig.*65. |
| 27 = 17 | 30 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 31 | $x^4+px^3+qx^2+rx=S.$ | |
| *Transp.* | 32 | $x^4+px^3+qx^2+rx-S=0.$ *Q.e.d.* in Biquad. ſi $-S$. | *Fig.*66. |
| 27 = 18 | 33 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 34 | $x^4+px^3+qx^2+rx=-S.$ | |
| *Transp.* | 35 | $x^4+px^3+qx^2+rx+S=0.$ *Q.e.d.* in Biquad. ſi $+S$. | *Fig.*67. |

*Illuſtrat.*

$$7\ \begin{Bmatrix}x^3 - \overset{p.}{24}x^2 + \overset{q.}{450}x - \overset{r.}{1775} = 0\\ x^3 - 2.4x^2 + 4.50x - 1.775 = 0\end{Bmatrix}$$

$NO = x = 5.$

$$8\begin{cases} \overset{p.}{} & \overset{q.}{} & \overset{r.}{} \\ x^3 + 24x^2 + 450x + 1775 = 0 \\ x^3 + 2.4x^2 + 4.50x + 1.775 = 0 \end{cases}$$

$$NO = -x = -5.$$

$$\begin{cases} \frac{p}{2} = 1.2 & \frac{p^2}{4} = 1.44 & \frac{p^3}{8} = 1.728 \\ \frac{p}{4} = 0.6. & \frac{p^2}{8} = 0.72. & \frac{p^3}{16} = 0.864. \end{cases}$$

*Central.*

$$+\frac{q}{2} = 2.25 \qquad\qquad +\frac{pq}{4} = 2.700$$

$$-\frac{p^2}{8} = 0.72 \qquad\qquad -\frac{r}{2} = 0.8875$$

$$-\frac{L}{2} = 0.5 \qquad\qquad -\frac{p^3}{16} = 0.864$$

$$1.22 \qquad\qquad -\frac{p}{4} = 0.6$$

$$b = 1.03 = AD. \qquad\qquad 2.3515$$

$$d = 0.3485 = DH.$$

Fig. 65.

$$13\begin{cases} \overset{p.}{} & \overset{q.}{} & \overset{r.}{} & \overset{s.}{} \\ x^4 - 20x^3 + 500x^2 - 496x - 11520 = 0 \\ x^4 - 2.0x^3 + 5.00x^2 - 0.496x - 1.1520 = 0 \end{cases}$$

$$\begin{cases} NO = x = 5.9 + \\ MO = -x = -4. \end{cases}$$

$$14\begin{cases} \overset{p.}{} & \overset{q.}{} & \overset{r.}{} & \overset{s.}{} \\ x^4 + 20x^3 + 500x^2 + 496x - 11520 = 0 \\ x^4 + 2.0x^3 + 5.00x^2 + 0.496x - 1.1520 = 0 \end{cases}$$

Fig. 66.

$$\begin{cases} MO = x = 4. \\ NO = -x = -5.9 + \end{cases}$$

$$\begin{cases} \frac{p}{2} = 1.0. & \frac{p^2}{4} = 1.00. & \frac{p^3}{8} = 1.000. \\ \frac{p}{4} = 0.5. & \frac{p^2}{8} = 0.50. & \frac{p^3}{16} = 0.500. \end{cases}$$

Gen-

*Central.*

$$+\frac{q}{2}=2.50 \quad\quad +\frac{pq}{4}=2.50$$

$$-\frac{p^2}{8}=0.50 \quad\quad -\frac{r}{2}=0.248$$

$$-\frac{L}{2}=0.5 \quad\quad -\frac{p^3}{16}=0.500$$

$$1.00 \quad\quad -\frac{p}{4}=0.5$$

$$b=1.50=AD. \quad\quad 1.248$$

$$d=1.252=DH.$$

Fig.66.

$$\begin{array}{l} \quad\quad p. \quad\quad q. \quad\quad r. \quad\quad s. \\ 15\left\{\begin{array}{l} x^4-24x^3+450x^2-2000x+1125=0 \\ x^4-2.4x^3+4.50x^2-2.000x+0.1125=0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO=x=5. \\ no=x=0.66 \textit{ ferè}. \end{array}\right.$$

$$\begin{array}{l} \quad\quad p. \quad\quad q. \quad\quad r. \quad\quad s. \\ 16\left\{\begin{array}{l} x^4+24x^3+450x^2+2000x+1125=0 \\ x^4+2.4x^3+4.50x^2+2.000x+0.1125=0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO=-x=-5. \\ no=-x=-0.66 \textit{ ferè}. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=1.2. & \frac{p^2}{4}=1.44. & \frac{p^3}{8}=1.728. \\ \frac{p}{4}=0.6. & \frac{p^3}{8}=0.72. & \frac{p^3}{16}=0.864. \end{array}\right.$$

Fig.67.

*Central.*

$$+\frac{q}{2}=2.25 \quad\quad +\frac{pq}{4}=2.700$$

$$-\frac{p^2}{8}=0.72 \quad\quad -\frac{r}{2}=1.000$$

$$-\frac{L}{2}=0.5 \quad\quad -\frac{p^3}{16}=0.864$$

$$1.22 \quad\quad -\frac{p}{4}=0.6$$

$$b=1.03=AD. \quad\quad 2.464$$

$$d=0.236=DH.$$

And

At manum de Tabulâ; quandoquidem methodum quâ ad Regulam Catholicam exquirendam usus sum, (quò impensiùs demiraretur Mathematicorum vulgus, & nonnullos altioris subsellii suspensos teneam) quasi occultum quoddam mysterium pressisse, in animo erat.

Apud Diophantum enim, aliosque quam plurimos, quà Veteranos quà Neotericos, sepiuscule artem celare (quod maximam artem autumant) consuetudinem invaluisse non ignoro. Et hoc quidem de industriâ fecisse statueram, ut Tyronibus (in quorum usum solum hæc exarata sunt, & quibus solis hâc in re consultum est) voluptatem illius, proprio Marte, investigandæ, non præripiam. Diù quidem multùmque animo revolvi, quid agerem; anxius hærebam, & quò me verterem, planè nesciebam. Tandèm verò (diebus haud paucis elapsis) suscepto consilio non stare, sed à proposito resilire decrevi: Et à sententiâ priore ideò decedere visum est, nempè quòd voluptatem quam à fontibus Geometricis haurire expectarent Tyrones, cruciatus, inter inquirendum (rebus etiam non semper auspicatò succedentibus), minimè compensaturos suspicabar. Ab illis igitur mihi gratias habitum iri persuasum habui, si eos tanto onere levarem, tantisque ex ambagibus & Mæandris manuducerem, methodum ipsam, quà Regulam Generalem ipse excogitavi & comperi, breviter perstringendo.

Sæpè numerò admirari soleo, quibus mediis, sive vestigiis in Æquationibus Cubicis & Biquadraticis (quibus secundus Terminus deest) construendis, insistebat Cartesius. At tantum non obstupui, saltem non satis mirari potui, quomodo egregio illo viro (quo nihil majus habet orbis Geometricus, tanto ingenii acumine & perspicacitate imbuto, ut omnes in sui admirationem meritò rapiat), tam portentosa & rara (ad hanc rem spectantia) perspicienti, etiam alia ejusdem farinæ, & perspectu quidem æque facilia non deprehendisse contigisset; nempè, quòd non æque Parabolæ cujusvis Diametrum ac Axem animadvertisset. Si enim à Puncto quovis intra vel extra Parabolam positione dato,

Cir-

And here I had determined to put a period to this Tract, intending to suppress the Method, by which I came in prospect of the *General Rule*, and to reserve it as some choice Secret; to the intent, that the meaner sort of *Mathematicians* might be rapp'd into admiration, and that I might detain some others of an higher form in suspence:

It being an usual thing with *Diophantus*, and very many other *Veterans* as well as *Neoterics*, to conceal their Art, which they look upon as the greatest point of Art. And this truly designedly I had determined to do, that I might not anticipate that pleasure, which *Tyro*'s (for whose use and sake only all is done what I have done) might, by their own industry, find in its search very much; and a long while concerned I was, and at a stand what to do, either to conceal, or discover it: At length (tho long first) I resolved not to stand to my former determinations, but to make a discovery; and that which swayed and prevailed with me most, was, That the pleasure which *Tyro*'s might expect whiles busied in its Inquest, I suspected might not make them a suitable compensation (things not always succeeding according to their expectation) for the pains they might sustain in its search. I conceived therefore I might do them an acceptable office, if I should ease them of so great a burden, and lead them out of those Mazes and Labyrinths, in which they might be toyled, by discovering that Method by which I my self found out the General Rule.

I have oftentimes wondered what Mediums *Des Cartes* used in the finding out of the construction of such Cubic and Biquadratic Equations, wherein the second Term is wanting: But I wondred much more, that so excellent a Man (than whom the Geometrical World hath none greater, one endued with so great sharpness of wit and perspicacity, that he deservedly becomes the wonder of all), seeing and finding such wonderful rare things as (touching this business) he did, should not have been so happy as to see and find other things also of the same nature, and altogether as obvious; *viz.* That he had not as well considered the Diameter of a Parabole, as its Axe; for if it had been his hap, from any Point within or without any Parabole given in Position, to have

Circulum tam per verticem Diametri cujuspiam positione similiter datæ, quàm per verticem Axis transeuntem, descripsisse; & Operationem quam ad Axem ad Diametrum applicuisse, sors dedisset; rem (de quâ quærimus) acu tetigisse, nempè Regulam Generalem ad Æquationes omnes Cubicas & Biquadraticas, quomodolibet affectas construendas persentiscere contigisset. Quid in causâ fuisset, quòd Diametrum non respexisse contigerit, me fugit; at verò, pace tanti viri, suspicari liceat; scilicet, V. CL. proprietatem quandam Parabolæ ad hanc rem apprimè spectantem, (ut non dicam, nescivisse) paulò minùs pensiculatè trutinâsse; quod si fecisset, dubio procul (si modò iisdem, quibus ipse usus sum, mediis innixus fuisset, cùm tam nova, & prius orbi inaudita protulit) Universalem Regulam nullo negotio indagâsset.

De

have described a Circle, passing as well through the Vertex of any Diameter, (given likewise in Position) as through the Vertex of its Axe, and had applied that Operation to the Diameter, which he did to the Axe, he could not have missed that mark which we aim at, *viz.* have found a Method for the construction of all Cubic and Biquadratic Equations, howsoever affected, as we have found. What the reason might be, he had no respect unto the Diameter, I know not; but (begging his pardon) I humbly conceive, This most excellent acute Man (I cannot say, was ignorant of, but) took not into his consideration a certain propriety of a Parabole, (than which none could so aptly have suited his design) which if it had been his hap so to have done, no doubt (he using the same Mediums which I am about to discover, when he amused the World with such rare Inventions, and things never before heard of) he could not but with greatest ease have made a full discovery of the *Universal Rule.*

## *De Regulâ Generali investigandâ.*

SI à puncto quolibet (puta H) positione dato, intra vel extra quamlibet Parabolam datam, & sub eodem Plano, describatur Circulus, transiens per Verticem $\left\{\begin{matrix}\text{Axis}\\\text{Diametri}\end{matrix}\right\}$ positione datæ, secans Parabolam, in 1, 2, 3, seu 4 Punctis; & abs ipsis demittantur ad $\left\{\begin{matrix}\text{Axem}\\\text{Diametrum}\end{matrix}\right\}$ Perpendiculares; non solum omnes Æquationum formulæ, quartum gradum non excedentium elucescent; sed ad ipsas Construendas Regulæ possint elici. Enimverò, *Fig.* $\left\{\begin{matrix}13\\50\end{matrix}\right.$

Si à Puncto H (positione dato,) demittatur ad $\left\{\begin{matrix}\text{Axem}\\\text{Diametrum}\end{matrix}\right\}$ Perpendicularis H D, erunt D H, A D, positione similiter datæ. *Fig.* $\left\{\begin{matrix}13\\50\end{matrix}\right.$

Et si à vertice Diametri A, (*Fig.* 50.) ducatur A B ∥ D H; erit B A positione data. *Fig.* 50.

Pro variâ positione Puncti H liquido constat, Punctum D, citra vel ultra $\left\{\begin{matrix}\text{Axis}\\\text{Diametri}\end{matrix}\right\}$ verticem cadere posse.

Omnes varias positiones Puncti H, supervacaneum esset exponere; unicam tantùm, instar omnium proferam, & in exemplum dabo; nempe.

Fingatur Punctum H, ad lævam $\left\{\begin{matrix}\text{Axis}\\\text{Diametri}\end{matrix}\right\}$ positione dari; & Punctum D citra eorundem vertices cadere. *Fig.* $\left\{\begin{matrix}13\\50\end{matrix}\right.$

## *The manner of finding out the General Rule.*

IF from any Point (as H) given in position, either within or without any given Parabole, and on the same Plane be described a Circle, passing through the Vertex of its $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ given in position, *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ Cutting the Parabole in, 1, 2, 3, or 4 Points; and from them be demitted Perpendiculars to the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ as all forms of Equations, not exceeding the fourth degree, so Rules for their Construction may easily be had. For,

If from the Point H, (given in position) be demitted H D, Perpendicular to the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ then will D H, *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ A D, be given in position.

And if from the Vertex of the Diameter (A) (*Fig.* 50.) be drawn A B ∽ D H, then will B A likewise be given in position. *Fig.* 50.

According to the various position of the Point H, it is evident, that the Point D, may happen below or above the Vertex of the $\left\{\begin{matrix}\text{Axe.}\\ \text{Diameter.}\end{matrix}\right\}$

It were needless to set down all the diverse positions of the Point H; I shall instance in one only, for all, *viz.*

Let the Point H, be supposed to be posited towards the left side of the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ and the Point D to *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ fall below either of their Vertex's.

## *SERIES* 1.

| | | | |
|---|---|---|---|
| | 1 | Lactus Rect. $= L$, dat. | |
| | 2 | × × × | |
| | 3 | $AD = b$ } positione data. | |
| | 4 | $DH = d$ | |
| 47, e 1 <br> Q. 3 + <br> Q. 4. | 5 | $AD^2 + DH^2 = HA^2$ <br> $b^2 + d^2 = HA^2$ | |
| Supp. | 6 | $NO = x.$ | *Fig.* 13. |
| | 7 | × × × | |
| Ob para. | 8 | $L \,.\, NO :: NO \,.\, AO.$ <br> $L \,.\, x :: x \,.\, \frac{x^2}{L} = AO.$ | |
| $8 \backsim 3$ | 9 | $AO \backsim AD = (DO, u)\ HP.$ <br> $\frac{x^2}{L} \backsim b = HP.$ | |
| ⊖ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} = HP^2$ | |
| 6 — 4 | 11 | $NO - (OP, u)\ DH = PN.$ <br> $x - d = PN.$ | |
| ⊖ | 12 | $d^2 + x^2 - 2dx = PN^2$ | |
| 47, e 1 <br> 10 + 12 | 13 | $HP^2 + PN^2 = HN^2$ <br> $b^2 + d^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} + x^2 - 2dx = HN^2$, reduc. | |
| 13 = 5 | 14 | $b^2 + d^2 + \frac{x^4}{L^2} - 2Lbx^2 - \frac{2L^2dx}{L^2} = HN^2 = (\S\, 5)\ b^2 + d^2 + \frac{L^2x^2}{L^2}$ | *Fig.* 13. |

S E-

## *SERIES* 1.

| | | | |
|---|---|---|---|
| | 1 | Lactus Rect. $= L$, given. | |
| | 2 | × × × | |
| | 3 | $AD = b$ } given in position. | |
| | 4 | $DH = d$ } | |
| 47, e 1 Q. 3 + Q. 4. | 5 | $\begin{cases} AD^2 + DH^2 = HA^2 \\ b^2 + d^2 = HA^2 \end{cases}$ | |
| Supp. | 6 | $MO = x$. | *Fig.* 13. |
| | 7 | × × × | |
| Ob para. | 8 | $\begin{cases} L \,.\, MO :: M \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{cases}$ | |
| 3 ∽ 8 | 9 | $\begin{cases} AD \backsim AO = (DO, u)\, HP. \\ b \backsim \frac{x^2}{L} = HP \end{cases}$ | |
| ☉ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} = HP^2$ | |
| 6 + 4 | 11 | $\begin{cases} MO + (OP, u)\, DH = PM. \\ x + d = PM. \end{cases}$ | |
| ☉ | 12 | $d^2 + x^2 + 2dx = PM^2$ | |
| 47, e 1 10 + 12 | 13 | $\begin{cases} HP^2 + PM^2 = HM^2 \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} + x^2 + 2dx = HM^2 \text{; reduced,} \end{cases}$ | |
| 13 = 5 | 14 | $b^2 + d^2 + \frac{x^4}{L^2} - 2Lbx^2 + \frac{2L^2dx}{L^2} = HM^2 = (\S\, 5)\, b^2 + d^2 + \frac{L^2x^2}{L^2}$ | *Fig.* 13. |

S E-

## *SERIES* 2.

| | | | |
|---|---|---|---|
| | 1 | Latus Rect. $= L$, dat. | |
| | 2 | $BA = a$ | |
| | 3 | $AD = b$ $\Big\}$ positione data. | *Fig.* 50. |
| | 4 | $DH = d$ | |
| 47, *e* 1 *Q.* 3. + *Q.* 4. | 5 | $\begin{cases} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = HA^2. \end{cases}$ | |
| *Supp.* | 6 | $NO = x.$ | *Fig.* 50. |
| 6 — 2 | 7 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - a = OR. \end{cases}$ | |
| *Ob para.* | 8 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - a \,.\, \frac{x^2}{L} - \frac{ax}{L} = AO. \end{cases}$ | |
| 3 ∽ 8 | 9 | $\begin{cases} AD \backsim AO = (DO, u)\ HP. \\ b - \frac{x^2}{L} + \frac{ax}{L} = HP. \end{cases}$ | |
| ⊙ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} + \frac{2abx}{L} = HP^2.$ | |
| 6 — 4 | 11 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ x - d = PN. \end{cases}$ | |
| ⊙ | 12 | $d^2 + x^2 - 2dx = PN^2.$ | |
| 47, *e* 1. 10 + 12 | 13 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ \left.\begin{matrix} b^2 + d^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} + x^2 \\ + \frac{2abx}{L} + 2dx. \end{matrix}\right\} = HN^2, \text{ reduc.} \end{cases}$ | |
| 13 = 5 | 14 | $\left.\begin{matrix} b^2 + d^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} \begin{matrix} + a^2 \\ - 2Lb \\ + L^2 \end{matrix}\frac{x^2}{L^2} \begin{matrix} + 2Lab \\ - 2L^2d \end{matrix}\frac{x}{L^2} \end{matrix}\right\} \begin{matrix} = HN^2 = \\ = (\S\, 5)\ b^2 + d^2. \end{matrix}$ | *Fig.* 50. |

Hinc

## SERIES 2.

| | | | |
|---|---|---|---|
| | 1 | Latus Rect. = L, given. | |
| | 2<br>3<br>4 | $\left.\begin{array}{l} BA = a \\ AD = b \\ DH = d \end{array}\right\}$ given in position. | *Fig.* 50. |
| 47, e 1<br>Q. 3. +<br>Q. 4. | 5 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = HA^2. \end{array}\right.$ | |
| *Supp.* | 6 | $MO = x.$ | *Fig.* 50. |
| 6 + 2 | 7 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + a = OR. \end{array}\right.$ | |
| *Ob para.* | 8 | $\left\{\begin{array}{l} L \;.\; MO \;::\; OR \;.\; AO. \\ L \;.\; x \;::\; x + a \;.\; \frac{x^2}{L} + \frac{ax}{L} = AO. \end{array}\right.$ | |
| 3 ∽ 8 | 9 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\, HP. \\ b - \frac{x^2}{L} - \frac{ax}{L} = HP. \end{array}\right.$ | |
| ⊖ | 10 | $b^2 + \frac{x^4}{L^2} + \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} - \frac{2abx}{L} = HP^2.$ | |
| 6 + 4 | 11 | $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ x + d = PM. \end{array}\right.$ | |
| ⊖ | 12 | $d^2 + x^2 + 2dx = PM^2.$ | |
| 47, e 1<br>10 + 12 | 13 | $\left\{\begin{array}{l} HP^2 + PM^2 = HM^2. \\ \left\{\begin{array}{l} b^2 + d^2 + \frac{x^4}{L^2} + \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} + x^2 \\ - \frac{2abx}{L} - 2dx. \end{array}\right\} = HM^2, \text{ reduce.} \end{array}\right.$ | |
| 13 = 5 | 14 | $b^2 + d^2 + \frac{x^4}{L^2} + \frac{2ax^3}{L^2} + \begin{array}{l} a^2x^2 - 2Labx \\ - 2Lb + \frac{2L^2d}{L^2} \\ + \frac{L^2}{L^2} \end{array} \left\{\begin{array}{l} = HM^2 = (S\ 5 \\ = b^2 + d^2. \end{array}\right.$ | *Fig.* 50. |

Hence

Hinc (per § 14, utriusque Seriei,) liquidò constat; si pro Homogeneo comparationis, fingatur.

f 1. $b^2+d^2=HA^2=$ Q. Rad. tum $b^2+d^2$ evanescere, residuamque Æquationem (Multiplicatam in $\frac{L^2}{x}$, ) ad Cubicam, (quâ quidem terminus secundus deficiet in primâ Serie; nullus verò in secundâ) deprimi. *Fig.* {13 50

g 2. $b^2+d^2\pm\frac{S}{L^2}=$ Q. Rad. tum $b^2+d^2$ etiam evanescere, residuamque Æquationem in quarto gradu subsistere, hoc est fore Biquadraticam; quâ, in primâ serie terminum secundum; nullum verò terminorum, in secundâ, deficere continget.

Quomodo autem in figurâ construi possit $b^2+d^2\pm\frac{S}{L^2}=$ Q. Rad. paucis expedire operæ erit pretium.

Describantur duo diversa paria Parabolarum, in quarum altero (pari,) (nempe, ut in *Fig.* 14, 15.) sint AD, DH similes & similiter positæ, ac in *Fig.* 13; in altero verò (nempe ut in *Fig.* 51. 52.) sint BA, AD, DH similes & similiter positæ, ac in *Fig.* 50. Jam,

1. Si à vertice {Axis / Diametri} erigatur ad AH Perpendicularis $AL=\sqrt{\frac{S}{L^2}}$; vel (quod perinde est,) si *Fig.* {14 51
h in linea AH, productâ utrinque, ex unâ parte sumatur
i $AI=L$, & ex alterâ parte $AK=\frac{S}{L^3}$, orietur $AL^2$
k $=\frac{S}{L^2}$.

Et

Hence it is manifeſt, (by § 14 of both Series's;) if for the Homogene of the compariſon, be ſuppoſed,

f 1. $b^2+d^2=HA^2=Q.$ Rad. then $b^2+d^2$ will vaniſh, and the remaining Equation be depreſſed (being Multiplied into $\frac{L^2}{x}$,) to a Cubick; in which the ſecond term will be wanting in the firſt Series; but none of the terms, in the ſecond. *Fig.* $\left\{\begin{matrix}13\\50\end{matrix}\right.$

g 2. $b^2+d^2\pm\frac{S}{L^2}=Q.$ Rad. then $b^2+d^2$ will vaniſh, (as before,) and the remaining Equation to ſubſiſt in the fourth degree, that is, will be a Biquadratic; where, in the firſt Series, the ſecond term, but in the ſecond Series, neither of the terms will happen to be wanting.

How $b^2+d^2\pm\frac{S}{L^2}$ may be made $=Q.$ Rad. I will briefly ſhew.

Let there be deſcribed two divers pairs of Paraboles; in the one of which (as in *Fig.* 14, 15.) let AD, DH be like, and a like poſited, as in *Fig.* 13; but in the other pair (as in *Fig.* 51. 52.) let BA, AD, DH be like, and a like poſited, as in *Fig.* 50. Now,

1. If from the Vertex of the $\left\{\begin{matrix}\text{Axe}\\\text{Diameter}\end{matrix}\right\}$ be erected to AH a Perpendicular $AL=\sqrt{\frac{S}{L^2}}$; or, (which is all one,) if in the line AH, both ways produced, be taken on the one ſide $AI=L$, and on the other ſide $AK=\frac{S}{L^3}$, then will $AL^2=\frac{S}{L^2}$. *Fig.* $\left\{\begin{matrix}14\\51\end{matrix}\right.$

h i k

m $\text{Et}\left\{\begin{aligned}&HA^2 + AL^2 = HL^2\\&b^2 + d^2 + \frac{S}{L^2} = \text{Q. Rad.}\end{aligned}\right\}$ q. e. f. 1. Fig. $\left\{\begin{matrix}14\\51\end{matrix}\right.$

2. Si Diametro A H, describatur Semicirculus, in quo, vel inscribatur $AZ = \sqrt{\frac{S}{L^2}}$; vel (quod idem est) summatur $AZ = AL$ (supra (k) inventæ,) orietur

n $AZ^2 = \frac{S}{L^2}$.

o $\text{Et}\left\{\begin{aligned}&AH^2 - AZ^2 = HZ^2\\&b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad.}\end{aligned}\right\}$ q. e. f. 2. Fig. $\left\{\begin{matrix}15\\52\end{matrix}\right.$

Series prima continuata.

$14 = \left\{\begin{matrix}m\\o\end{matrix}\right.$ 15 $\frac{x^4}{L^2} * \begin{matrix}-2Lbx^2\\+\frac{L^2x^2}{L^2}\end{matrix} - \frac{2L^2dx}{L^2} = \pm\frac{S}{L^2}$; in $L^2$, &c.

*Transp.* 16 $x^4 * \begin{matrix}-2Lbx^2\\+L^2.\end{matrix} - 2L^2dx\ (\mp S) = 0.$

$14 = 5$ 17 $\frac{x^4}{L^2} * \begin{matrix}-2Lbx^2\\+\frac{L^2x^2}{L^2}\end{matrix} - \frac{2L^2dx}{L^2} = 0$; in $\frac{L^2}{x}$. $14 = f$

$\times\frac{L^2}{x}$ 18 $x^3 * \begin{matrix}-2Lbx\\+L^2x.\end{matrix} - 2L^2d = 0.$

Series secunda continuata.

$14 = \left\{\begin{matrix}m\\o\end{matrix}\right.$ 15 $\left.\frac{x^4}{L^2} = \frac{2ax^3}{L^2} + \begin{matrix}a^2x^2\\-2Lb\\+\frac{L^2}{L^2}\end{matrix} + \begin{matrix}2Labx\\-\frac{2L^2d}{L^2}\end{matrix}\right\} = \pm\frac{S}{L^2}$; in $L^2$, &c.

*Transp.* 16 $\left.x^4 - 2ax^3 + \begin{matrix}a^2x^2\\-2Lb\\+L^2\end{matrix} + \begin{matrix}2Labx\\-2L^2d\end{matrix}\ (\mp S)\right\} = 0.$

| | | | |
|---|---|---|---|
| 47, e 1 $\S + k$ | *m* | And $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2 \\ b^2 + d^2 + \frac{S}{L^2} = Q.\,Rad. \end{array}\right\}$ w. w. d. | *Fig.* $\left\{\begin{array}{l}14\\51\end{array}\right.$ |
| | *n* | 2. If on the Diameter A H be a Semicircle defcrib'd, and in it, either infcribed $AZ = \sqrt{\frac{S}{L^2}}$, or (which is the fame thing,) made $AZ = AL$ above ($\S$ k) found, then will $AZ^2 = \frac{S}{L^2}$. | |
| | *o* | And $\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2 \\ b^2 + d^2 - \frac{S}{L^2} = Q\,Rad. \end{array}\right\}$ w. w. d. | *Fig.* $\left\{\begin{array}{l}15\\52\end{array}\right.$ |

### The firft Series continued.

| | | | |
|---|---|---|---|
| $14 = \left\{\begin{array}{l}m\\o\end{array}\right.$ | 15 | $\frac{x^4}{L^2} * \begin{array}{l} -2Lbx^2 \\ +\frac{L^2x^2}{L^2} \end{array} + \frac{2L^2dx}{L^2} = (\pm\frac{S}{L^2};)$ in $L^2$, and | |
| *Tranfp.* | 16 | $x^4 * \begin{array}{l} -2Lbx^2 \\ +L^2x^2. \end{array} + 2L^2dx\ (\mp S) = o.$ | |
| $14 = 5$ | 17 | $\frac{x^4}{L^2} * \frac{\begin{array}{l} -2Lbx^2 \\ +L^2x^2 \end{array}}{L^2} + \frac{2L^2dx}{L^2} = o;$ in $\frac{L^2}{x}$. | $14 = f$ |
| $*\frac{L^2}{x}$ | 18 | $x^3 * \begin{array}{l} -2Lbx \\ +L^2x. \end{array} + 2L^2d = o.$ | |

### The fecond Series continued.

| | | |
|---|---|---|
| $14 = \left\{\begin{array}{l}m\\o\end{array}\right.$ | 15 | $\left.\frac{x^4}{L^2} + \frac{2ax^3}{L^3} \begin{array}{l} +a^2x^2 \\ \frac{-2Lb \atop +L^2}{L^2} \end{array} \begin{array}{l} -2Labx \\ +\frac{2L^2d}{L^2} \end{array}\right\} = \pm\frac{S}{L^2};$ in $L^2$, and |
| *Tranfp.* | 16 | $x^4 + 2ax^3 \begin{array}{l} +a^2x^2 \\ -2Lb \\ +L^2 \end{array} \begin{array}{l} -2Labx \\ +2L^2d \end{array} (\mp S) = o.$ |

$14 = 5$ | 17 | $$\left.\begin{array}{l}\frac{x^4}{L^2} - \frac{2ax^3}{L^2} + a^2x^2 + 2Labx \\ \quad - 2Lb - \frac{2L^2d}{L^2} \\ \quad + \frac{L^2}{L^2}\end{array}\right\} = 0;\ \text{in}\ \frac{L^2}{x}.$$ $14 = f$

$\times \frac{L^2}{x}$ | 18 | $$\left.\begin{array}{l}x^3 - 2ax^2 + a^2x + 2Lab \\ \quad - 2Lb - 2L^2d \\ \quad + L^2.\end{array}\right\} = 0.$$

### *In Æquationibus Construendis.*

*Observ.* 1 1. Cubicis, ubi terminus secundus $\left\{\begin{array}{l}\text{deerit}\\ \text{aderit}\end{array}\right\}$ Circulum quidem per Verticem $\left\{\begin{array}{l}\text{Axis.}\\ \text{Diametri.}\end{array}\right\}$ *Fig.* $\left\{\begin{array}{l}13\\ 50\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}13\\ 50\end{array}\right.$

2. Biquadraticis verò per extremum quidem L, rectæ $AL = \sqrt{\frac{S}{L^2}}$ (à vertice $\left\{\begin{array}{l}\text{Axis}\\ \text{Diametri}\end{array}\right\}$ ad HA Perpendiculariter erectæ, si in Æquatione habeatur $- S$. *Fig.* $\left\{\begin{array}{l}14\\ 51\end{array}\right.$

Per extremum verò Z, rectæ $AZ = \frac{S}{L^2}$, in Semicirculo (cujus diameter sit HA,) à vertice $\left\{\begin{array}{l}\text{Axis}\\ \text{Diametri}\end{array}\right\}$ inscriptæ, si ibi habeatur $+ S$ oportere transire, Quæ omnia ab Æquationibus (16, 18) patent. *Fig.* $\left\{\begin{array}{l}15\\ 52\end{array}\right.$

*Observ.* 2 Si Æquationis cujuslibet propositæ signa, quæ quidem in secundo & quarto termino reperiantur, mutentur, (quæ in tertio verò invariatis,) diversa quidem à propositâ evadet Æquatio, easdem cum ipsâ habens radices; quarum quæ in unâ Æquatione sunt veræ, in alterâ evadent falsæ, & contra; & consequentèr utrique construendæ ipsa eadem Regula (quæcunque fuerit) inservire necesse est.

Patet

$14=5$ | 17 | $$\left.\begin{matrix}\frac{x^4}{L^2}+\frac{2ax^3}{L^2}+a^2x^2-2Labx \\ -2Lb+\frac{2L^2d}{L^2} \\ +\frac{L^2}{L^2}\end{matrix}\right\}=0;\ \text{in}\ \frac{L^2}{x}.$$ $14=f$

$\times\frac{L^2}{x}$ | 18 | $$\left.\begin{matrix}x^3+2ax^2+a^2x-2Lab \\ -2Lb+2L^2d \\ +L^2.\end{matrix}\right\}=0.$$

## *In the Constructions of Equations.*

*Observ.* 1. 1. Cubic, where the second term is $\left\{\begin{matrix}\text{wanting} \\ \text{not wanting}\end{matrix}\right\}$ the Circle, must pass through the Vertex of the $\left.\begin{matrix}\text{Axe.} \\ \text{Diameter.}\end{matrix}\right\}$ *Fig.* $\left\{\begin{matrix}13 \\ 50\end{matrix}\right.$ *Fig.* $\left\{\begin{matrix}13 \\ 5.\end{matrix}\right.$

2. But in Biquadraticks, through the Point L, of the Right-line $AL=\surd\frac{S}{L^2}$ (from the Vertex of the $\left\{\begin{matrix}\text{Axe} \\ \text{Diameter}\end{matrix}\right\}$ to H A Perpendicularly erected,) if in the Equation be had $-$ S. *Fig.* $\left\{\begin{matrix}14 \\ 15\end{matrix}\right.$

But through the Point Z, of the Right-line $AZ=\surd\frac{S}{L^2}$, inscribed in a Semicircle (whose Diameter is H A,) from the Vertex of the $\left\{\begin{matrix}\text{Axe} \\ \text{Diameter}\end{matrix}\right\}$ if be had $+$ S. All which is evident, from Equations 16, 18, *&c.* *Fig.* $\left\{\begin{matrix}15 \\ 52\end{matrix}\right.$

*Observ.* 2. If of any Equation proposed the signs be changed, which are found in the second and fourth terms, (those in the third remaining unchanged,) another Equation distinct from that proposed will appear, having the same Roots with it; of which, those which are true in the one Equation, will be false in the other; and consequently the same Rule (whatsoever it be,) will serve for both their Constructions.

This

Patet ex comparatione Æquationum 16, 18, in primâ Serie, cum 16, 18, in secundâ: A quantitate enim S, fieri non potest, ut immutetur (quam non ingreditur) Regula; quemadmodum fusius infrà patebit.

---

*De investigandâ Regulâ unicuique Æquationum formulæ Construendæ inserviente.*

Si utriusque Seriei Æquationem 16, 18, (suprà inventam,) cum aliâ simplici assumptâ simili comparemus, & unumquemque terminum illius, correspondenti termino hujus adæquari fingamus; Regula Centralis sub involucris Co-efficientium latitans, è latebris eruere cogetur.

---

*Primo*, Æquationem 16 vel 18 (ubi secundus terminus deficere contingit,) suprà inventam, cum aliâ simili & æquali assumptâ (quâ similiter secundus terminus deficit) conferamus; nempe.

$$\begin{array}{r|l} 16 & \left\{\begin{array}{l} x^4 * \begin{array}{l}-2Lb\\+L^2\end{array} x^2 - 2L^2dx\,(\mp S) = 0 \\ x^3 * \begin{array}{l}-2Lb\\+L^2\end{array} x - 2L^2d = 0. \end{array}\right\} \text{ inventam.} \\ 18 & \end{array}$$

Comparandam cum,

$$\begin{array}{r|l} 19 & \left\{\begin{array}{l} x^4 * \mp qx^2 - qx\,(\mp S) = 0 \\ x^3 * \mp qx - r = 0. \end{array}\right\} \text{ assumptâ.} \\ 20 & \end{array}$$

Primò fingamus,

$$\begin{array}{r r|l} 20 & 21 & \left\{\begin{array}{l} x^3 * - qx - r = 0. \\ x^4 * - qx^2 - rx\,(\mp S) = 0 \end{array}\right\} \text{ assumptam.} \\ 19 & & \end{array}$$

Ex

This appears, by comparing the 16 and 18 Equations in the first Series, with the 16 and 18 in the second: For it is impossible that the quantity S, should make any change in that Rule, of which it is no Part; as hereafter shall more largely appear.

---

*Of finding out a Rule, serving for the Construction of each several form of Equations.*

If we compare the 16 or 18 Equations of both Series's, with another assumed simple and alike; and suppose each term of that, to be equal to its Correspondent term of this; the Central Rule, which lies hid under the Covert of the Co-efficients, will easily be detected.

---

*First*, Let us compare the 16 or 18 Equation before found, (where the second term happens to be wanting) with another like and equal assumed, in which likewise the second term is wanting: *viz.*

$$\left.\begin{array}{ll} 16 & x^4 * \begin{array}{l}-2Lbx^2\\+L^2\end{array} + 2L^2dx\,(\mp S) = 0 \\ 18 & x^3 * \begin{array}{l}-2Lbx\\+L^2.\end{array} + 2L^2d = 0 \end{array}\right\}\text{ fonnd.}$$

To be compared with,

$$\left.\begin{array}{ll} 19 & x^4 * \mp qx^2 + rx\,(\mp S) = 0 \\ 20 & x^3 * \mp qx + r = 0. \end{array}\right\}\text{ assumed.}$$

First, suppose we,

$$21\ \left.\begin{array}{l} x^3 * - qx + r = 0. \\ x^4 * - qx^2 + rx\,(\mp S) = 0 \end{array}\right\}\text{ assumed.}$$

It

Ex hypothesi liquet.

| | | |
|---|---|---|
| $18=21$ | 22 | $-2Lb+L^2=-q$, ejus correspondenti. |
| *Transp.* | 23 | $L^2+q=2Lb$. |
| $\frac{23}{2L}$ | 24 | $\frac{L}{2}+\frac{q}{2L}=b=AD$, in Axem. |
| $18=21$ | 25 | Pari jure, $-2L^2d=-r$ ejus correspond. |
| $\frac{25}{2L^2}$ | 26 | g° $\frac{r}{2L^2}=d=DH$, ad Axem $\perp$. |

*Consectar.* 1.

Si proponeretur Æquatio construenda,

| | | |
|---|---|---|
| 21 | 27 | $\left\{\begin{array}{l} x^3 * -qx\mp r=0. \\ x^4 * -qx^2\mp rx\ (\mp S)=0. \end{array}\right\}$ Fig. $\left\{\begin{array}{l}13\\14\\15\end{array}\right.$ |
| 24<br>26 | | Orietur $\left\{\begin{array}{l} \frac{L}{2}+\frac{q}{2L}=b=AD \\ \frac{r}{2L^2}=d=DH \end{array}\right\}$ Central. |

Secundo, fingamus,

| | | |
|---|---|---|
| 20<br>19 | 28 | $\left\{\begin{array}{l} x^3 * +qx-r=0. \\ x^4 * +qx^2-rx\ (\mp S)=0 \end{array}\right\}$ assumptam. |

Ex hypothesi pate,

| | | |
|---|---|---|
| $18=28$ | 29 | $-2Lb+L^2=+q$, ejus correspondenti. |
| *Transp.* | 30 | $L^2-q=2Lb$. |
| $\frac{30}{2L}$ | 31 | $\frac{L}{2}-\frac{q}{2L}=b=AD$, in Axem. |
| $18=29$ | 32 | Æquo jure, $-2L^2d=-r$, ejus corresp. |
| $\frac{32}{2L^2}$ | 33 | g° $\frac{r}{2L^2}=d=DH$, $\perp$ ad Axem. |

*Con-*

It is evident by our ſuppoſition.

18=20 22 $-2Lb+L^2=-q$, its correſpondent.

*Tranſp.* 23 $L^2+q=2Lb$.

24 $\frac{L}{2}+\frac{q}{2L}=b=AD$, upon the Axe.

18=20 25 By parity of reaſon, $+2L^2d=+r$.

$\frac{25}{2L^2}$ 26 $g^\circ$, $\frac{r}{2L^2}=d=DH$, $\perp$ to the Axe.

*Conſectar.* I.

If the Equation propoſed ſhould be,

21 27 $\left\{\begin{array}{l} x^3 * -qx\pm r=0. \\ x^4 * -qx^2\pm rx\ (\mp S)=0. \end{array}\right\}$ *Fig.* $\left\{\begin{array}{l} 13 \\ 14 \\ 15 \end{array}\right.$

24, 26 Then will $\left\{\begin{array}{l} \frac{L}{2}+\frac{q}{2L}=b=AD \\ \frac{r}{2L^2}=d=DH \end{array}\right\}$ Central.

Secondly, ſuppoſe we,

20, 19 28 $\left\{\begin{array}{l} x^3 * +qx+r=0. \\ x^4 * +qx^2+rx\ (\mp S)=0 \end{array}\right\}$ aſſumed.

It is manifeſt by our hypotheſis,

18=28 29 $-2Lb+L^2=+q$, its correſpondent.

*Tranſp.* 30 $L^2-q=2Lb$.

$\frac{30}{2L}$ 31 $\frac{L}{2}-\frac{q}{2L}=b=AD$, upon the Axe.

18=28 32 By the ſame reaſon, $+2L^2d=+r=$ its correſp.

$\frac{32}{2L^2}$ 33 $g^\circ$, $\frac{r}{2L^2}=d=DH$, $\perp$ to the Axe.

*Consectar.* 2.

Si proponeretur Æquatio construenda,

28 34 $\begin{cases} x^3 * + q x \mp r = o. \\ x^4 * + q x^2 \mp r x (\mp S) = o \end{cases}$ *Fig.* 16. *Fig.* $\begin{cases} 17 \\ 18 \end{cases}$

31 Orietur $\begin{cases} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \frac{r}{2L^2} = d = DH \end{cases}$ Central.

33

Quæ duo Consectaria quartum Classem complectuntur.

---

*Secundò*, Fingamus in Æquatione 16 vel 18 inventâ, & in 19, 20 assumptâ, tertium terminum etiam deficere; nempè,

16 35 $x^4 * \left(\begin{matrix} -2Lbx^2 \\ +L^2 \end{matrix}\right) - 2L^2 d x (\mp S) = o$

18 36 $x^3 * (= q x) - 2L^2 d = o.$ inventam.

Comparandum cum,

19 37 $x^4 ** \quad - r x (\mp S) = o$

20 38 $x^3 ** \quad - r = o.$ assumptâ.

Quandoquidem ex hypothesi supponimus $q = o$, g°,

35=37 39 $-2Lb + L^2 = o$, ejus corresp. ($L^2 = 2Lb$, vel)

*Transp.* 40 $L = 2b$.

$\frac{40}{2}$ 41 $\frac{L}{2} = b = AD$, in Axem.

35=37 42 Est autem $\frac{r}{2L^2} = d = DH$, $\perp$ ad Axem; (vide § 25, 26; vel § 32, 33.)

*Con-*

*Consectar.* 2.

If an Equation proposed, were,

28 34 $\left\{\begin{array}{l} x^3 * + qx \pm r = 0. \\ x^4 * + qx^2 \pm rx\,(\mp S) = 0 \end{array}\right\}$ *Fig.* 16. *Fig.* $\left\{\begin{array}{l}17\\18\end{array}\right.$

31 Then will $\left\{\begin{array}{l} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.
33

Which two Consectaries comprehend the fourth Class.

---

*Secondly*, Suppose in the 19 and 20 Equations assumed, the third term (or q) to be also wanting, *viz.*

16 35 $\left\{\begin{array}{l} x^4 * \left(\begin{array}{l}-2Lbx^2\\+L^2\end{array}\right) + 2L^2dx\,(\pm S) = 0 \\ x^3 * \left(\begin{array}{l}-2Lbx\\+L^2\end{array}\right) + 2L^2d = 0. \end{array}\right\}$ found,
18 36

To be compared with,

19 37 $\left\{\begin{array}{l} x^4 * * \quad + rx\,(\mp S) = 0. \\ x^3 * * \quad + r = 0. \end{array}\right\}$ assumed.
20 38

Forasmuch as is supposed $q = 0$; g°,

35=37 39 $-2Lb + L^2 = 0$, its correspondent,

*Transp.* 40 $(L^2 = 2Lb$, or$)\ L = 2b$.

$\frac{40}{2}$ 41 $\frac{L}{2} = b = AD$, on the Axe.

And $\frac{r}{2L^2} = d = DH$ (as before § 25, 26; and § 32, 33.)



Con-

*Consectar*. 3.

Si proponeretur Æquatio construenda,

38 | 43 $$\begin{cases} x^3 * * \mp r = o. \\ x^4 * * \mp r x\ (\mp S) = o. \end{cases}$$ Fig. 10.

37 | Fig. 11, 12

41 | 42 $$\text{Orietur} \begin{cases} \frac{L}{2} = b = A\,D \\ \frac{r}{2L^2} = d = D\,H \end{cases} \text{Central.}$$

Quod tertiam Æquationum Classem complectitur.

---

*Tertiò*, Fingamus, in Æquatione 16, vel 18 inventâ, & in 19, 20 assumptâ, quartum terminum (præter secundum) deficere; nempè,

16 | 44 | 18 | 45 $$\left.\begin{array}{l} x^4 * \begin{array}{l}- 2Lbx^2 \\ + L^2\end{array} (- 2L^2 dx)\ (\mp S) = o \\ x^3 * \begin{array}{l}- 2Lbx \\ + L^2\end{array} (- 2L^2 dx) = o, \text{ hoc est} \\ x^2 * \begin{array}{l}- 2Lbx \\ + L^2\end{array} \begin{array}{l}- 2Lb \\ + L^2\end{array} = o. \end{array}\right\} \text{inventum},$$

Comparandum cum,

19 | 46 | 20 | 47 $$\left.\begin{array}{l} x^4 * \mp q\, x^2 * (\mp S) = o. \\ x^2 * \mp q = o. \end{array}\right\} \text{assumptâ.}$$

Primò fingamus,

47 | 46 | 48 $$\left.\begin{array}{l} x^2 * - q = o. \\ x^4 * - q\, x^2 * (\mp S) = o. \end{array}\right\} \text{assumptam.}$$

Ex

*Consectar.* 3.

If the Equation to be made, were,

38 43 $\left\{\begin{array}{l} x^3 * * \pm r = 0. \end{array}\right.$ Fig. 10.

37 $\left.\begin{array}{l} x^4 * * \pm rx\ (\mp S) = 0 \end{array}\right\}$ Fig. $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

41 Then will $\left\{\begin{array}{l} \frac{L}{2} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

42

Which Comprehends all the Equations of the third Clafs.

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*Thirdly*, Suppofe (in the 16 or 18 Equation found, and in the 19, 20 affumed) the fourth term (befide the fecond) to be wanting; *viz.*

16 44 $\left\{\begin{array}{l} x^4 * \begin{array}{l} -2Lb \\ +L^2 \end{array} x^2 (+2L^2 dx)\ (\mp S) = 0 \end{array}\right.$

18 45 $\left.\begin{array}{l} x^2 * \left.\begin{array}{l} -2Lb \\ +L^2 \end{array}\right\} = 0. \end{array}\right\}$ found,

To be compared with,

19 46 $\left\{\begin{array}{l} x^4 * \mp q x^2 * (\mp S) = 0 \end{array}\right.$

20 47 $\left.\begin{array}{l} x^2 * \pm q = 0. \end{array}\right\}$ affumed.

Firft fuppofe,

47 48 $\left\{\begin{array}{l} x^2 * - q = 0. \end{array}\right.$

46 $\left.\begin{array}{l} x^4 * - q x^2 * (\mp S) = 0. \end{array}\right\}$ affumed.

It

Ex hypothesi patet,

44 = 48 49 $-2Lb + L^2 = -q$, ejus correspond.

*Transp.* 50 $L^2 + q = 2Lb$.

$\frac{50}{2L}$ 51 g°, $\frac{L}{2} + \frac{q}{2L} = b = AD$, in Axem.

Ex hypothesi etiam liquet,

44 = 48 52 $o = q = 2L^2 d$, ejus correspond.

52 53 g°, $d = DH = o$; adeòque, Punctum D & H in Axem co-incidere.

*Consectar.* 4.

Si proponeretur Æquatio construenda,

47 / 46 54 $\left\{\begin{array}{l} x^2 * - q = o. \\ x^4 * - q x^2 * (\mp S) = o. \end{array}\right\}$ *Fig.* 5. *Fig.* $\left\{\begin{array}{l} 6. \\ 7. \end{array}\right.$

51 / 53 Orietur $\left\{\begin{array}{r} \frac{L}{2} + \frac{q}{2L} = b = AD \\ o = d = DH \end{array}\right\}$ Central.

Secondò finge,

47 / 46 55 $\left\{\begin{array}{l} x^2 * + q = o. \\ x^4 * + q x^2 * (-S) = o. \end{array}\right\}$ assumptam.

Ex hypothesi patet,

44 = 55 56 $-2Lb + L^2 = +q$.

*Transp.* 57 $L^2 - q = 2Lb$.

$\frac{57}{2L}$ 58 $\frac{L}{2} - \frac{q}{2L} = b = AD$, in Axem.

59 Est autem $d = DH = o$, *&c.* (ut suprà § 53.)

Con-

It is evident by our supposition,

44=48 49 $-2Lb+L^2=-q$.
Transp. 50 $L^2+q=2Lb$.
$\frac{50}{2L}$ 51 g°, $\frac{L}{2}+\frac{q}{2L}=b=AD$, on the Axe.

It appears likewise by the supposition,

44=48 52 $o=q=2L^2d$, its correspondent,
52 53 g°, $d=DH=o$, and g°, the Point D and H, to be co-incident on the Axe.

*Consectar.* 4.

If the Equation to be made, were,

47
46 54 $\left\{\begin{array}{l} x^2 \ast - q = o \\ x^4 \ast - qx^2 \ast (\mp S) = o \end{array}\right\}$ *Fig.* 5. *Fig.* $\left\{\begin{array}{l} 6. \\ 7. \end{array}\right.$

51
53 Then will $\left\{\begin{array}{r} \frac{L}{2}+\frac{q}{2L}=b=AD \\ o=d=DH \end{array}\right\}$ Central.

Secondly, suppose,

47
46 55 $\left\{\begin{array}{l} x^2 \ast + q = o. \\ x^4 \ast + qx^2 \ast (-S) = o \end{array}\right\}$ assumed.

By supposition it appears,

44=55 56 $-2Lb+L^2=+q$.
Transp. 57 $L^2-q=2Lb$.
$\frac{57}{2L}$ 58 g°, $\frac{L}{2}-\frac{q}{2L}=b=AD$, on the Axe.
59 Now $d=DH=o$, &c. (as above § 53.)

Con-

*Consectar.* 5.

Si proponeretur Æquatio construenda,

47 60 $* \begin{cases} x^2 * + q = o. \\ x^4 * + q x^2 * (-S) = o. \end{cases}$ *Fig.* 8.
46

58 Orietur $\begin{cases} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ o = d = DH \end{cases}$ Central.
59

Quæ quidem duo ultima Consectaria omnes Æquationum secundi Classis formulas complectuntur.

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*Quartò*, Fingamus in Æquatione assumptâ § 19, (cui Æquatio § 16 adæquari supponitur) quantitates, tum q & r (præter p) deficere, nempè

61 $x^4 * * * - S = o$ assumpta.

16 = 61 62 Ex hypothesi $q = o$; g°, $-2Lb + L^2 = o$, ejus correspondenti; &

*Transp.* 63 $(L^2 = 2Lb$, vel$)$ $L = 2b$.

$\frac{63}{2}$ 64 g°, $\frac{L}{2} = b = AD$, in Axem.

65 Est autem $d = DH = o$, *&c.* (ut suprà, § 53.)

*Consectar.* 6.

61 66 Si proponeretur Æquatio construenda, *Fig.* 4.

$x^4 * * * - S = o$.

64 Orietur $\begin{cases} \frac{L}{2} = b = AD \\ o = d = DH \end{cases}$ Central.
65

Consectaria 5 & 6, primam Æquationum Classem concludunt.

Hactenùs de Regulâ Centrali investigandâ, ubi secundus terminus continuò deficit.

*Secundò*,

*Consectar.* 5.

If the Equation proposed to be made,

47 | 60 be $\left\{\begin{array}{l} x^2 * + q = o. \\ x^4 * + q x^2 * (-S) = o. \end{array}\right\}$ *Fig.* 8.
46

58 | Then will $\left\{\begin{array}{l} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \quad o = d = DH \end{array}\right\}$ Central.
59

Which two last Consectaries comprehend all the forms of Equations in the second Class.

---

*Fourthly*, Suppose in the assumed Equation § 19, (to which the 16th. is supposed to be equal,) the quantities both q and r (besides p) to be wanting, *viz.*

61 $x^4 * * * - S = o$ assumed.

16 = 61 | 62 By supposition $q = o$; g°, $-2Lb + L^2 = o$, its correspondent; and

Transp. | 63 $(L^2 = 2Lb$, or$)$ $L = 2b$.

$\frac{63}{2}$ | 64 g°, $\frac{L}{2} = b = AD$, on the Axe.

65 Now $d = DH = o$, *&c.* (as above § 53.)

*Consectar.* 6.

If the Equation proposed to be made be,

61 | 66 $x^4 * * * - S = o$. *Fig.* 4.

64 | Then will $\left\{\begin{array}{l} \frac{L}{2} = b = AD \\ o = d = DH \end{array}\right\}$ Central.

65 | The 5 and 6 Consectaries conclude the first Classis.

Hitherto of finding out the Central Rule, for the Construction of all Equations, where the second term is wanting.

*Secundò*, Æquationem suprà inventam § 16, 18, in secundâ Serie, (ubi secundus terminus non deficere contingit) cum aliâ simplici & Æquali assumptâ, (quâ similiter secundus terminus aderit) inter se conferamus: nempè,

16 67
$$\left.\begin{array}{l} x^4 - 2ax^3 + a^2x^2 + 2Labx\,(\mp S) = 0. \\ \qquad\qquad\;\; - 2Lb \;\; - 2L^2d \\ \qquad\qquad\;\; + L^2 \end{array}\right\} \text{invent.}$$

18 68
$$\left.\begin{array}{l} x^3 - 2ax^2 + a^2x + 2Lab \\ \qquad\qquad\; - 2Lb - 2L^2d \\ \qquad\qquad\; + L^2 \end{array}\right\} = 0.$$

Comparandam cum,

69 70
$$\left.\begin{array}{l} x^4 - px^3 \pm qx^2 \pm rx\,(\mp S) = 0. \\ x^3 - px^2 \pm qx \pm r = 0. \end{array}\right\} \text{assumptâ.}$$

67 = 69 71 Manifestò apparet, $2a = p$, ejus corresp. ideòque

71 72 $\frac{p}{2} = a = BA$: (& $\frac{p^2}{4} = a^2$.)

*Observ.* 3 Hinc, si in Æquatione Construendâ propositâ, quartum gradum non excedente, reperiatur quantitas p; oportere ordinatim ad Axem continuò applicari rectam $BA = \frac{p}{2}$, occurrentem Parabolæ in B & A; à quorum alterutro Puncto concursûs (ab A puta) agi rectam A y Axi Parallelam: vel (quod perinde est) à vertice Axis, erigi debere à dextrâ Parabolæ ad Axem Perpendicularem $E = \frac{p}{4}$; & ex E, (actâ E A ipsi Axi Parallelâ, donec occurrat Parabolæ in A) duci A B ipsi a E Parallelam: quo fiet, distantiam Diametri ab Axe reperiri continuò: quod quidem animadvertisse operæ forsan erit pretium.

Jam

*Secondly*, Compare we the Equation above found, § 16, or 18, in the second Series, (where the second term happens not to be wanting) with another like and equal assumed, in which likewise the second term is had: *viz.*

16 67 $$x^4+2ax^3+\begin{array}{l}a^2\\-2Lb+2L^2d\\+L^2\end{array}x^2-2Labx\,(\mp S)=0.$$

18 68 $$\left.\begin{array}{l}x^3+2ax^2+a^2x-2Lab\\\qquad\qquad\quad-2Lb+2L^2d\\\qquad\qquad\quad+L^2\end{array}\right\}=0.$$

found

To be Compared with,

69 $$x^4+px^3\pm qx^2\mp rx\,(\mp S)=0.$$

70 $$x^3+px^2\pm qx\mp rq=0.$$

assumed.

*Observ.* 3 Hence, if in any Equation, whose Construction is required, not exceeding the fourth degree, be found the quantity p, it follows, that there ought always be ordinately applied to the Axe, a Right-line (as) $BA=\frac{p}{2}$, meeting the Parabole in B and A; from either of which Points of meeting (as suppose from A) must be drawn A y Parallel to the Axe; or (which is the same) from the Vertex of the Axe, must (towards the Right-side of the Parabole) be erected the Perpendicular $aE=\frac{p}{4}$; and from E, (E A being drawn Parallel to the Axe, 'till it meets the Parabole in A) must be drawn A B, Parallel to the said a E; by which means, the distance of the Diameter from the Axe will always be had; which to have taken notice of, may perhaps be worth the while.

Jam ſi in locum (a) (§ 67, 68.) ſubſtituatur ejus valor, nempè $\frac{p}{2}$; & in locum $a^2$, ejus valor, nempè $\frac{p^2}{4}$, orietur Æquatio nova Æquationi § 67, vel 68, inventæ ad æquata, nempè,

73 $$\left.\begin{cases} x^4 - p x^3 + \frac{p^2}{4} x^2 + L p b x \ (\mp S) = 0. \\ \qquad\qquad - 2 L b - 2 L^2 d \\ \qquad\qquad + L^2 \\ x^3 - p x^2 + \frac{p^2}{4} x + L p b = 0. \\ \qquad\qquad - 2 L b - 2 L^2 d \\ \qquad\qquad + L^2 \end{cases}\right\} \text{(inventa)}$$

Comparanda cum,

69 / 70 — 74 $$\left.\begin{cases} x^4 - p x^3 \pm q x^2 \pm r x \ (\mp S) = 0. \\ x^3 - p x^2 \pm q x \pm r = 0. \end{cases}\right\} \text{aſſumptâ.}$$

Primò finge,

75 $$\left.\begin{cases} x^4 - p x^3 - q x^2 - r x \ (\mp S) = 0. \\ x^3 - p x^2 - q x - r = 0. \end{cases}\right\} \text{aſſumptam.}$$

73 = 75 | 76 Conſtat, $\frac{p^2}{4} - 2 L b + L^2 = - q$ ejus correſp.

*Tranſp.* | 77 $L^2 + \frac{p^2}{4} + q = 2 L b.$

$\frac{77}{2L}$ | 78 $\frac{L}{2} + \frac{p^2}{8 L} + \frac{q^2}{2L} = b = AD$, in Diametrum.

73 = 75 | 79 Item, $+ L p b - 2 L^2 d = - r$, ejus correſpond. & ſi in locum b ſubſtituatur ejus valor (ſuprà inventus) ſc.

Now, if (in § 67, 68) in the place of (a) be ſubſtituted its valor, *viz.* $\frac{p}{2}$; and in the place $a^2$, its valor, *viz.* $\frac{p^2}{4}$, will ariſe a new Equation, equal to that found, § 67, or 68, *viz.*

73 $$\left\{\begin{array}{l} x^4 + p x^3 + \frac{p^2}{4} x^2 - L p b x \; (\mp S) = 0. \\ \qquad\qquad\;\; - 2 L b + 2 L^2 d \\ \qquad\qquad\;\; + L^2 \\ x^3 + p x^2 + \frac{p^2}{4} x - L p b = 0. \\ \qquad\qquad\;\; - 2 L b + 2 L^2 d \\ \qquad\qquad\;\; + L^2 \end{array}\right\} \text{found}$$

To be compared with,

69, 70 | 74 $$\left\{\begin{array}{l} x^4 + p x^3 \pm q x^2 \mp r x \; (\mp S) = 0. \\ x^3 + p x^2 \pm q x \mp r = 0. \end{array}\right\} \text{aſſumed.}$$

First ſuppoſe,

75 $$\left\{\begin{array}{l} x^4 + p x^3 - q x^2 + r x \; (\mp S) = 0. \\ x^3 + p x^2 - q x + r = 0. \end{array}\right\}$$

73 = 75 | 76 It is manifeſt, $+ \frac{p^2}{4} \, 2 L b + L^2 = - q$ its correſp.

*Tranſp.* | 77 $L^2 + \frac{p^2}{4} + q = 2 L b.$

$\frac{77}{2L}$ | 78 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, on the Diameter.

73 = 75 | 79 Again, $- L p b + 2 L^2 d = + r$ its correſp.

*Tranſp.* | 80 $L p b + r = 2 L^2 d$; and if in the place of b, be ſubſtituted its valor, (above found) *viz.*

| | | |
|---|---|---|
| 78 | | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$, fiet |
| 78, 79 | 80 | $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}-2L^2d=-r$. |
| *Tranſp.* | 81 | $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}+r=2L^2d$. |
| $\frac{81}{2L^2}$ | 82 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH$; $\perp$ ad Diam. |
| | | *Conſectar.* 1. |
| | | Si proponeretur Æquatio conſtruenda, |
| 75 | 83 | $\left\{\begin{array}{l} x^4 \mp px^3 - qx^2 \mp rx(\mp S)=0. \\ x^3 \mp px^2 - qx \mp r = 0. \end{array}\right\}$ *Fig.* $\left\{\begin{array}{l}42\\43\end{array}\right.$ *Fig.* 41. |
| 78<br>82 | | Orietur $\left\{\begin{array}{l} \frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD \\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH \end{array}\right\}$ Centr. |
| | | Secundò, Finge inventam § 73, comparandam |
| 74 | 84 | cum $\left\{\begin{array}{l} x^3-px^2-qx+r=0 \\ x^4-px^3-qx^2+rx(\mp S)=0. \end{array}\right\}$ aſſumptà. |
| 73 = 84 | 85 | Patet, $+\frac{p^2}{4}-2Lb+L^2=-q$ ejus correſpond. |
| *Tranſp.* | 86 | $L^2+\frac{p^2}{4}+q=2Lb$. |
| $\frac{86}{2L}$ | 87 | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD$; in Diametrum. |
| 73 = 84 | 88 | Æquo jure, $+Lpb-2L^2d=+r$, ejus correſp. |
| *Tranſp.* | 89 | $Lpb-r=2L^2d$.<br>Si in locum b, ſubſtituatur ejus valor (ſuprà inventus) ſc. |
| 87<br>89 | 90 | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$; fiet, $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}-r=2L^2d$. |
| $\frac{90}{2L^2}$ | 91 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=d=DH$; $\perp$ ad Diametr. |

*Con-*

78 $\frac{L}{2}+\frac{p^2}{8L}+\frac{q^2}{2L}$: Then will be

78, 79 81 $\frac{L^2 p}{2}+\frac{p^3}{8}+\frac{pq}{2}+r=2L^2 d.$

$\frac{81}{2L^2}$ 82 $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH$; $\perp$ to the Diam.

*Consectar.* 1.

If the Equation proposed to be made,

75 83 be $\left\{\begin{array}{l} x^4 \pm px^3 - qx^2 \pm rx \left\{\begin{array}{l}(\mp S)\\(\pm S)\end{array}\right\} = o. \\ x^3 \mp px^2 - qx \pm r = o. \end{array}\right.$ Fig. $\left\{\begin{array}{l}42\\43\end{array}\right.$ Fig. $\left\{\begin{array}{l}42\\43\end{array}\right.$ Fig. 41.

78 / 82 Then will $\left\{\begin{array}{l} \frac{L}{2}+\frac{p}{8L}+\frac{q}{2L}=b=AD \\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH \end{array}\right\}$ Cent.

Secondly, suppose § 73, found to be compared

74 84 with $\left\{\begin{array}{l} x^3+px^2-qx-r=o. \\ x^4+px^3-qx^2-rx\ (\mp S)=o. \end{array}\right\}$ assumed.

73 = 84 85 It is plain; $+\frac{p^2}{4}-2Lb+L^2=-q$, its corresp.

*Transp.* 86 $L^2+\frac{p^2}{4}+q=2Lb.$

$\frac{86}{2L}$ 87 $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD$; on the Diameter.

73 = 84 88 Also, $-Lpb+2L^2 d=-r$, its correspond.

*Transp.* 89 $Lpb-r=2L^2 d.$

If in the place of b, be substituted its valor (above found)

87, 89 90 *viz.* $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$; then will be, $\frac{L^2 p}{2}+\frac{p^3}{8}+\frac{pq}{2}-r=2L^2 d.$

$\frac{90}{2L^2}$ 91 $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=d=DH$; $\perp$ to the Diam.

*Con-*

*Consectar.* 2.

Si proponeretur Æquatio construenda, *Fig.* 44.

84 92 $\begin{cases} x^3 \mp px^2 - qx \pm r = 0. \\ x^4 \mp px^3 - qx^2 \pm rx \left\{ \begin{matrix} (\mp S) \\ (\mp S) \end{matrix} \right\} = 0, \text{ fiet} \end{cases}$ *Fig.* $\left\{ \begin{matrix} 45 \\ 46 \end{matrix} \right.$ *Fig.* $\left\{ \begin{matrix} 45 \\ 46 \end{matrix} \right.$

87 / 91

$$\left. \begin{cases} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH \end{cases} \right\} \text{Central.}$$

Tertiò, Finge inventam § 73, comparandam

74 93 cum $\left. \begin{cases} x^3 - px^2 + qx + r = 0. \\ x^4 - px^3 + qx^2 + rx\,(\mp S) = 0. \end{cases} \right\}$ assumptâ.

$73 = 93$ 94 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = +q$, ejus correspond.

*Transp.* 95 $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{95}{2L}$ 96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; in Diametrum.

$73 = 93$ 97 Item, $+Lpb - 2L^2 d = +r$ ejus correspond.

*Transp.* 98 $Lpb - r = 2L^2 d$; si in locum b, substituatur ejus valor (suprà inventus) nempè,

96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; fiet,

98 99 $\frac{L^2 p}{2} + \frac{p^3}{8} - \frac{pq}{2} - r = 2L^2 d.$

$\frac{99}{2L^2}$ 100 $\frac{p^2}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH$; $\perp$ ad Diametr.

*Con-*

*Consectar.* 2.

If the Equation, whose Construction is desired, be — Fig. 44.

84 92 $\left\{\begin{array}{l} x^3 \pm px^2 - qx \mp r = 0. \\ x^4 \pm px^3 - qx^2 \mp rx (\mp S) = 0. \end{array}\right\}$ then will — Fig. $\left\{\begin{array}{l}45\\46\end{array}\right.$ Fig. $\left\{\begin{array}{l}45\\46\end{array}\right.$

87, 91 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

Thirdly, Suppose § 73 found, to be compared

74 93 with $\left\{\begin{array}{l} x^3 + px^2 + qx - r = 0. \\ x^4 + px^3 + qx^2 - rx (\mp S) = 0. \end{array}\right\}$ assumed.

73 = 93 94 It is evident, $\frac{p^2}{4} - 2Lb + L^2 = +q$, its corresp.

*Transp.* 95 $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{95}{2L}$ 96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; on the Diameter.

73 = 93 97 Also, $- Lpb + 2L^2 d = - r$ its correspond.

*Transp.* 98 $Lpb - r = 2L^2 d$; if in the place of b, be substituted its valor (before found) *viz.*

96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; then will be,

98 99 $\frac{L^2 p}{2} + \frac{p^3}{8} - \frac{pq}{2} - r = 2L^2 d.$

$\frac{99}{2L^2}$ 100 $\frac{p^2}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH$; $\perp$ to the Diam.

 *Con-*

*Consectar.* 4.

Si construenda proponeretur Æquatio, *Fig.* 59.

102 | 110
$$\left\{\begin{array}{l} x^3 \mp p x^2 + q x \mp r = o. \\ x^4 \mp p x^3 + q x^2 \mp r x \left\{\begin{array}{l} (\mp S) \\ (\mp S) \end{array}\right\} \begin{array}{l} = o \\ = o \end{array} \end{array}\right\} \text{ fiet,}$$
*Fig.* $\left\{\begin{array}{l} 60 \\ 61 \end{array}\right.$ *Fig.* $\left\{\begin{array}{l} 60 \\ 61 \end{array}\right.$

105
$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD.$$
109
$$\frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$
Central.

Quæ quidem quatuor Confectaria octavum Classem complectuntur.

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*Secundò*, Finge, in Æquatione 73 inventâ, & in 74 assumptâ, quartum terminum deficere; & 73 comparari,

$$x^3 \mp p x^2 \mp q x = o, \text{ hoc est}$$

111
$$\text{cum} \left\{\begin{array}{l} x^2 \mp p x \mp q = o. \\ x^4 \mp p x^3 \mp q x^2 * (\mp S) = o. \end{array}\right\} \text{ assumptâ.}$$

Primò finge,

111 | 112
$$\left\{\begin{array}{l} x^2 - p x - q = o. \\ x^4 - p x^3 - q x^2 * (\mp S) = o. \end{array}\right\} \text{ assumptâ.}$$

73 = 112 | 113 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = -q$, ejus correspond.

*Transp.* | 114 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{114}{2L}$ | 115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, in Diametrum.

Æquo

## Consectar. 4.

If the Equation proposed to be made Fig. 59.

102 110 be $\left\{\begin{array}{l} x^3 \pm p x^2 + q x \pm r = 0. \\ x^4 \pm p x^3 + q x^2 \pm r x \left\{\begin{array}{l}(\mp S)\\(\pm S)\end{array}\right\} = 0; \end{array}\right.$ then Fig. $\left\{\begin{array}{l}60\\61\end{array}\right.$ Fig. $\left\{\begin{array}{l}60\\61\end{array}\right.$

105
109

$$\text{will}\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH \end{array}\right\}\text{Cent.}$$

Which four Consectaries comprehend the eighth Classe of Æquations.

---

*Secondly*, Suppose in the 73d. Æquation found, and in the 74th. assumed, the fourth term to be wanting; and the 73d. to be compared.

$$x^3 \pm p x^2 \mp q x = 0;\ \text{that is}$$

111 $$\text{with}\left\{\begin{array}{l} x^2 \pm p x \mp q = 0. \\ x^4 \pm p x^3 \mp q x^2 * (\mp S) = 0. \end{array}\right\}\text{assumed.}$$

First, suppose,

111 112 $$\left\{\begin{array}{l} x^2 + p x - q = 0. \\ x^4 + p x^3 - q x^2 * (\mp S) = 0. \end{array}\right\}\text{assumed.}$$

73 = 112 113 It is evident, $\frac{p^2}{4} - 2Lb + L^2 = -q$; its corresp.

*Transp.* 114 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{114}{2L}$ 115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, on the Diameter.

By

Æquo jure; Quoniam supponimus quartum terminum, nempè r, deficere, hoc est $r = 0$;

73=112 116 $g^{\circ},\ +Lpb - 2L^2d = 0$, ejus correspond. &

*Transp.* 117 $(Lpb = 2L^2d$, vel$)\ pb = 2Ld$. Si igitur in locum b, substituatur ejus valor (suprà inventus) nempè,

115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L}$, orietur,

117 118 $\frac{Lp}{2} + \frac{p^3}{8L} + \frac{pq}{2L} = 2Ld$; &

$\frac{118}{2L}$ 119 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH$; $\perp$ ad Diametr.

*Consectar.* 5.

Si proponeretur Æquatio construenda,

*Fig.* 34.

112 120 $\left.\begin{cases} x^2 \mp px - q = 0. \\ x^4 \mp px^3 - qx^2 * \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} = 0. \end{cases}\right\}$ Orietur, *Fig.* $\begin{cases} 35 \\ 36 \end{cases}$ *Fig.* $\begin{cases} 35 \\ 36 \end{cases}$

115
119
$\left.\begin{cases} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH \end{cases}\right\}$ Central.

Secundò, finge,

111 121 $\left.\begin{cases} x^2 - px + q = 0. \\ x^4 - px^3 + qx^2 * (\mp S) = 0. \end{cases}\right\}$ assumptâ.

73=121 122 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = +q$, ejus corresp.

*Transp.* 123 $L^2 + \frac{p^2}{4} - q = 2Lb$.

$\frac{123}{2L}$ 124 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; in Diametrum.

Item,

116 By the same reason, forasmuch as the fourth term (*viz.* r) is supposed to be wanting; that is $r=0$;

73=112 116 g°, $-Lpb+2L^2d=0$, its correspondent; *i. e.*

Transp. 117 ($Lpb=2L^2d$ or) $pb=2Ld$: If therefore in the place of b, be substituted its valor (above found,) *viz.*

$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$: Then will be,

117 118 $\frac{Lp}{2}+\frac{p^3}{8L}+\frac{pq}{2L}=2Ld$; and

$\frac{118}{2L}$ 119 $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}=d=DH$; $\perp$ to the Diameter.

*Consectar.* 5.

If the Equation to be effected, were

112 120 $\left\{\begin{array}{l} x^2\pm px-q=0. \\ x^4\pm px^3-qx^2 * \left\{\begin{array}{c}(\mp S)\\(\mp S)\end{array}\right\}=0;\end{array}\right.$ then will be  Fig. 34. Fig. $\left\{\begin{array}{l}35\\36\end{array}\right.$ Fig. $\left\{\begin{array}{l}35\\36\end{array}\right.$

115 / 119 $\left.\left\{\begin{array}{l}\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD\\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}=d=DH\end{array}\right.\right\}$ Central.

Secondly, suppose,

111 121 $\left.\left\{\begin{array}{l}x^2+px+q=0.\\ x^4+px^3+qx^2 * (\mp S)=0.\end{array}\right.\right\}$ assumed.

73=121 122 It is evident $\frac{p^2}{4}-2Lb+L^2=+q$, its corresp.

Transp. 123 and $L^2+\frac{p^2}{4}-q=2Lb$.

$\frac{123}{2L}$ 124 g°, $\frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=b=AD$, on the Diameter.

Again,

Item, Quoniam ex hypothesi $r = o$;

73 = 121 125 $g^{o}$, $+ L p b - 2 L^{2} d = o$, ejus correspond. &

126 ($L p b = 2 L^{2} d$ vel) $p b = 2 L d$. Si igitur in locum $b$, substituatur ejus valor, nempè

124 $\frac{L}{2} + \frac{p^{2}}{8 L} - \frac{q}{2 L}$; fiet

126 127 $\frac{L p}{2} + \frac{p^{3}}{8 L} - \frac{p q}{2 L} = 2 L d$; &

$\frac{127}{2 L}$ 128 $\frac{p}{4} + \frac{p^{3}}{16 L^{2}} - \frac{p q}{4 L^{2}} = d = DH$; $\perp$ ad Diametrum.

*Consectar.* 6.

Sit Æquatio construenda proposita,

121 129 $\begin{cases} x^{2} \mp p x + q = o. \\ x^{4} \mp p x^{3} + q x^{2} * \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} = o; \end{cases}$ fiet *Fig.* 37 *Fig.* $\begin{cases} 38 \\ 39 \end{cases}$

124 / 128 $\begin{cases} \frac{L}{2} + \frac{p^{2}}{8 L} \backsim \frac{q}{2 L} = b = AD \\ \frac{p}{4} + \frac{p^{3}}{16 L^{2}} \backsim \frac{p q}{4 L^{2}} = d = DH \end{cases}$ Central. *Fig.* $\begin{cases} 38 \\ 39 \end{cases}$

Consectaria 5 & 6, septimum Classem Æquationum complectuntur.

*Tertiò*, Finge in Æquatione 73 inventâ, & in 74 assumptâ, tertium terminum deficere; & 73 comparari.

130 cum $\begin{cases} x^{3} \mp p x^{2} * \pm r = o. \\ x^{4} \mp p x^{3} * \pm r x (\mp S) = o \end{cases}$ assumptâ.

Primò

Again, by the hypothesis $r = o$,

73=121 | 125 | $g^{o}, -Lpb + 2L^{2}d = o$, its correspond. And
126 | $(Lpb = 2L^{2}d$ or$)$ $pb = 2Ld$. If then in the place of $b$, be substituted its valor, *viz.*

124 | $\frac{L}{2} + \frac{p^{2}}{8L} + \frac{q}{2L}$; then will

126 | 127 | $\frac{Lp}{2} + \frac{p^{3}}{8L} + \frac{pq}{2L} = 2Ld$; and

$\frac{127}{2L}$ | 128 | $\frac{p}{4} + \frac{p^{3}}{16L^{2}} - \frac{pq}{4L^{2}} = d = DH$; $\perp$ to the Diameter.

*Consectar.* 6.

If the Equation to be made, were

121 | 129 | $$\begin{cases} x^{2} \pm px + q = o. \\ x^{4} \pm px^{3} + qx^{2} * \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} = o; \end{cases}$$ then will Fig. 37, Fig. {38, 39}

Fig. {38, 39}

124 | 128 | $$\left.\begin{cases} \frac{L}{2} + \frac{p^{2}}{8L} \backsim \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^{3}}{16L^{2}} \backsim \frac{pq}{4L^{2}} = d = DH \end{cases}\right\}$$ Central.

The 5th. and 6th. Consectaries comprehend the seventh Class.

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*Thirdly*, Suppose in the 73d. Equation, found, and in the 74th. assumed, the third term to be wanting; and the 73d. to be compared

130 | with $$\left.\begin{cases} x^{3} \pm px^{2} * \mp r = o. \\ x^{4} \pm px^{3} * \mp rx(\mp S) = o \end{cases}\right\}$$ assumed.

Primò finge,

130 131 $\left.\begin{cases} x^3 - px^2 * + r = 0 \\ x^4 - px^3 * + rx^2 \ (\mp S) = 0 \end{cases}\right\}$ assumptâ.

Constat, Quoniam supponitur $q = 0$;

73=131 132 g°, $\frac{p^2}{4} - 2Lb + L^2 = 0$, ejus correspond.

*Transp.* 133 & $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{133}{2L}$ 134 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$, in Diametrum.

73=131 135 Liquet etiam $+ Lpb - 2L^2d = + r$, ejus corresp.

*Transp.* 136 g°, $Lpb - r = 2L^2d$. Si itaque in locum b, substituatur ejus valor, nempè,

134 $\frac{L}{2} + \frac{p^2}{8L}$; fiet,

136 137 $\frac{L^2p}{2} + \frac{p^3}{8} - r = 2L^2d$.

$\frac{137}{2L^2}$ 138 $\frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = DH$; in Diametrum.

*Consectar.* 7.

Sit Æquatio construenda, *&c.*

*Fig.* 25.

131 139 $\left.\begin{cases} x^3 \mp px^2 * \mp r = 0 \\ x^4 \mp px^3 * \mp rx \begin{cases} (\mp S) \\ (\pm S) \end{cases} \end{cases}\right\} = 0$; Orietur *Fig.* $\begin{cases}26\\27\end{cases}$ *Fig.* $\begin{cases}26\\27\end{cases}$

134 / 138 $\left.\begin{cases} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH \end{cases}\right\}$ Central.

Secundò,

**First, suppose,**

130 131 $\left\{\begin{array}{l} x^3 + px^2 * - r = 0. \\ x^4 + px^3 * rx\,(\mp S) = 0. \end{array}\right\}$ assumed.

It is evident, in as much as is supposed $q = 0$;

73=131 132 $g^{\circ}$, $+\frac{p^2}{4} - 2Lb + L^2 = 0$; its correspondent,

*Transp.* 133 and $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{133}{2L}$ 134 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$, on the Diameter.

73=131 135 It is evident also, $-Lpb + 2L^2d = -r$, its corresp.

*Transp.* 136 $g^{\circ}$, $Lpb - r = 2L^2d$. If therefore in the place of $b$, be substituted its valor, *viz.*

134 $\frac{L}{2} + \frac{p^2}{8L}$; then will,

136 137 $\frac{L^2p}{2} + \frac{p^3}{8} - r = 2L^2d$.

$\frac{137}{2L^2}$ 138 $\frac{p^2}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = DH$; on the Diameter.

*Consectar.* 7.

Let this Equation be to be made, *viz.*

*Fig.* 25.

131 139 $\left\{\begin{array}{l} x^3 \pm px^2 * \mp r = 0. \\ x^4 \pm px^3 * \mp rx \left\{\begin{array}{l}(\mp S) \\ (\mp S)\end{array}\right\} = 0; \end{array}\right.$ then will *Fig.* $\left\{\begin{array}{l}26\\27\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}26\\27\end{array}\right.$

134
138 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.



Secondly,

Secundo, finge,

130 140 $\left\{\begin{array}{l} x^3 - px^2 * - r = 0 \\ x^4 - px^3 * - rx\,(\mp S) = 0 \end{array}\right\}$ assumptam.

73=104 Liquet, (per § 132, 133, 134, )

134 141 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; in Diametrum.

73=140 142 Item, $+ Lpb - 2L^2 d = - r$, ejus correspond.

*Transp.* 143 g°, $Lpb + r = 2L^2 d$; si igitur in locum b, substituatur ejus valor, nempe,

141 $\frac{L}{2} + \frac{p^2}{8L}$; fiet,

143 144 $\frac{L^2 p}{2} + \frac{p^3}{8} + r = 2L^2 d$.

$\frac{144}{2L^2}$ 145 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH$; $\perp$ ad Diametr.

*Consectar.* 8.

Esto hæc Æquatio construenda, sc.

$\left\{\begin{array}{l} x^3 \mp px^2 * \mp r = 0 \\ x^4 \mp px^3 * \mp rx \left\{\begin{array}{l} (\mp S) \\ (\pm S) \end{array}\right\} = 0 \end{array}\right.$; fiet *Fig.* 31. *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$ *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$

140 146

141 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

145

Confectaria 7 & 8, Classem sextum Complectuntur.

Quartò,

**Secondly, ſuppoſe,**

130 140 $$\left.\begin{cases} x^3 + px^2 * + r = 0. \\ x^4 + px^3 * + rx\ (\mp S) = 0. \end{cases}\right\}\text{ aſſumed.}$$

73=140 It is plain (by § 132, 133, 134, that)

134 141 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; on the Diameter.

73=140 142 Alſo, $- Lpb + 2L^2d = + r$, its correſpond.

*Tranſp.* 143 $g^o$, $Lpb + r = 2L^2d$; if then, in the place of b, be ſubſtituted its valor, *viz.*

141 $\frac{L}{2} + \frac{p^2}{8L}$; then will

143 144 $\frac{L^2p}{2} + \frac{p^3}{8} + r = 2L^2d.$

$\frac{144}{2L^2}$ 145 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH$; $\perp$ on the Diameter.

*Conſectar.* 8.

**Let this Equation be to be made,** *viz.*

140 146 $$\begin{cases} x^3 \pm px^2 * \pm r = 0. \\ x^4 \pm px^3 * \pm rx \begin{Bmatrix} (\mp S) \\ (\pm S) \end{Bmatrix} = 0; \text{ then will} \end{cases}$$ *Fig.* 31. *Fig.* $\begin{cases}32\\33\end{cases}$

*Fig.* $\begin{cases}32\\33\end{cases}$

141, 145, $$\left.\begin{cases} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH \end{cases}\right\}\text{ Central.}$$

The 7th. and 8th. Conſectaries comprehend the ſixth Claſs of Equations.

Fourthly,

*Quartò*, Finge in Æquatione 73, inventâ, & in 74 assumptâ, tertium & quartum terminos deficere, & 73 comparari, cum

$$x^3 \mp p x^2 * * = 0; \text{ h e,}$$

147 $\left\{\begin{array}{l} x \mp p = 0 \\ x^4 \mp p x^3 * * (\mp S) = 0 \end{array}\right\}$ assumptâ.

Finge,

147 148 $\left\{\begin{array}{l} x - p = 0 \\ x^4 - p x^3 * * (\mp S) = 0 \end{array}\right\}$ assumptam.

Ex hypothesi, est $q = 0$,

73=148 149 g°, $\frac{p^2}{4} - 2Lb + L^2 = 0$, ejus corresp.

*Transp.* 150 & $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{150}{2L}$ 151 ideóque $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; in Diametrum.

Item, ex hypothesi est $r = 0$,

73=148 152 g°, $+ Lpb - 2L^2d = 0$, ejus correspond. &

*Transp.* 153 ($Lpb = 2L^2d$, vel) $pb = 2Ld$; in locum $b$, substituatur igitur ejus valor; nempè,

151 $\frac{L^2}{2} + \frac{p^2}{8L}$, & orietur

153 154 $\frac{Lp}{2} + \frac{p^3}{8L} = 2Ld$, &

$\frac{154}{2L}$ 155 $\frac{p^2}{4} + \frac{p^3}{16L^2} = d = DH$; $\perp$ ad Diametrum.

*Consectar.*

*Fourthly*, In the 73d. Æquation found, and in the 74th. assumed, suppose the 3d. and 4th. terms to be wanting, and the 73d. to be compared with,

147 $\left\{\begin{array}{l} x \pm p = 0 \\ x^4 \pm p x^3 * * (\mp S) = 0 \end{array}\right\}$ assumed.

Suppose,

147 148 $\left\{\begin{array}{l} x + p = 0 \\ x^4 + p x^3 * * (\mp S) = 0 \end{array}\right\}$ assumed.

By supposition, $q = 0$;

73=148 149 $g^o$, $\frac{p^2}{4} - 2Lb + L^2 = 0$, its correspondent,

*Transp.* 150 and $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{150}{2L}$ 151 and $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; on the Diameter.

Also, by supposition is $r = 0$;

73=148 152 $g^o$, $-Lpb + 2L^2 d = 0$, its correspondent, and

*Transp.* 153 $(Lpb = 2L^2 d$, or$)$ $pb = 2Ld$; in the place of $b$, let be substituted its valor, *viz.*

151 $\frac{L}{2} + \frac{p^2}{8L}$ and will be,

153 154 $\frac{Lp}{2} + \frac{p^3}{8L} = 2Ld$; and

$\frac{154}{2L}$ 155 $\frac{p}{4} + \frac{p^3}{16L^2} = d = DH$; $\perp$ to the Diameter.

*Consectar.*

*Consectar.* 9.

Finge hanc Æquationem construendam, nempè, *Fig.* 22.

147 156 $\left\{\begin{array}{l} x \mp p = 0. \\ x^4 \mp p x^3 * * \left\{ \begin{array}{l} (\mp S) \\ (\pm S) \end{array} \right\} = 0 \end{array}\right.$ ; fiet, *Fig.* $\left\{\begin{array}{l}23\\24\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}23\\24\end{array}\right.$

151 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$

155 $\frac{p}{4} + \frac{p^3}{16L^2} = d = DH$ Central.

Quod ultimum Consectarium quintum Classem complectitur.

---

E quibus omnibus Consectariis generalitèr observandum est:

*Observ.* 4 Regulæ Centralis quantitates & signa ita omninò determinari oportere, quemadmodum in iis, quæ ad ipsam (suprà, *pag.* 8.) annotavimus, sunt exposita.

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2/6

## *FINIS.*

### *Consectar*. 9.

Suppose this Æquation proposed to be made, *viz.* *Fig*.22.

147 156 $\left\{\begin{array}{l} x \pm p = 0. \\ x^4 \pm p\,x^3 * * \left\{\begin{array}{l}(\mp S)\\(\mp S)\end{array}\right\} = 0; \end{array}\right.$ then will *Fig*. {23 24 *Fig*. {23 24

151 155 $$\left.\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} = d = DH \end{array}\right\} \text{Central.}$$

Which last Consectary completes the 5th. Class.

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From all which Consectaries, it may be generally observed:

*Observ*.4 That the quantities and signs of the Central Rule, must be altogether so determined, as they are exposed in those things, which we have noted before, in the 8th. *pag.*

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*THE END.*

## *Some Books Printed for, and Sold by* Robert Clavel, *at the Sign of the* Peacock *in* St. Paul's Church-yard.

THE Annals of King *James*, and King *Charles* the First, Containing a faithful and impartial Account of the Great Affairs of State and Transactions of Parliaments in *England*, in *Folio*. Wherein several material Passages Relating to the late Civil Wars (not mentioned in former Histories) are made known; in particular, some of Mr. *Rushorth*'s Mistakes and Omissions. And first the Case of the Devorce of the Earl of *Essex* from his Countess, which had so great Influence on the ensuing Troubles, Related from the Original Proceedings in that Court.

2. The True Cause of the Troubles in our Church, *viz.* The Connivance of some Church-men at the Dissenters from the Government of the Church, as Established by Law, and the Favour found at Court from great Persons there.

3. King *James* not in so much Influenced by *Gondamore*, as is Related by Mr. *Rushworth*.

4. The Three Estates in Parliament, who they were, in King *James*'s Speech in Parliament, 1620.

5. An Authentick and Impartial Account of the beginning of the Troubles in *Scotland*, and the Wars which ensued.

6. The True State of our late Civil Wars, their Beginnings, Causes, who the Aggressors, &c. The rest are too large to take notice here, but may be seen in the Preface.

*Vare-*

*Varenius*'s *Geography* in *Folio*, *English*, Illustrated with many Copper Cuts.

Dr. *Willis*'s Works in *Folio*, *English*.

The History of the *Irish Rebellion*, traced from many preceding Acts to the Grand Eruption, the 23d. of *October*, 1641. and thence pursued to the *Act of Settlement*, 1662.

Tracts Written by *John Selden* of the Inner-Temple, Esq; and Translated by the Eminent Dr. *A. L.* The 1st. *Jani Anglorum facies altera*, with large Notes thereupon. 2ly, *Englands Epinomis*. 3ly, Of the Original of Ecclesiastical Jurisdictions of Testaments. The 4th, Of the Disposition or Administration of intestate Goods.

Mr. *Scrivener*'s Body of Divinity.

Dr. *Cumber* on the *Liturgy* in *Folio*.

Mr. *Sam*'s *Britannia*. *Ogleby*'s History of *Affrica*, *Asia*, and *America*.

Bishop of St. *Davids*'s Vindication of the Bishops Rights to Vote in Capital Cases —— his seasonable Corrective

The Compleat Catalogue to the end of *Easter Term*, 1684.

The Bishop of *Lincoln*'s Observations, and Animadversions on *Pope Pius* the V. his *Bull* against Queen *Elizabeth*: Whereunto is Annexed the *Bull* of *Pope Paul* the III. against King *Henry* the VIII.

Dr. *Cumber*'s Vindication of the Divine Right of Tyths.

Bishop of *Cork*'s Perswasive to all Protestants.

Religion and Loyalty supporting each other, in Vindication of the Loyal Addressors.

Bishop of St. *David's* *Billa Vera*; or Argument of *Ignoramus* —— his short way to a lasting Settlement, and Answer to *Sidney's Speech* —— his Advice to a sound Protestant and Proselyte of *Rome* call'd back.

Three Sermons of Dr. *Standishes*.

Two of Mr. *Richard Werge* of *New-Castle*.

One Sermon of Dr. *Morice* before the King.

Two of Dr. *Dixons*'s, Prebend of *Rochester*.

Dr. *Ward*'s Sermon of *Blandford*.

*Ogleby*'s *Esop* in *English*, adorn'd with 160 Sculptures.

A Discourse of Natural and Moral Impotency.

Bishop of St. *David*'s Answer to *Melius Inquirendum* —— his Answer to the *Protestant Reconciler*.

*Brown*'s Treatise of Preternatural Tumours.

*Mocket*'s Tractatus de politia Eccles. Anglicanæ.

The Reduction of *Ireland* to the Crown of *England*.

*Smith*'s Rhetorick, the Fifth Edition.

*Humphrey*'s Resolution of Conscience.

Dr. *Byan*'s Eight Sermons, Preached before His Majesty in his Exile.

Friendly Conference between a Minister and a Quaker, Two parts.

Dr. *Duport*'s *Poems*. *Seneca* with *Farnaby*. *Skicard*'s Hebrew Grammar.

*Esop*'s Fables, Greek and Latin.

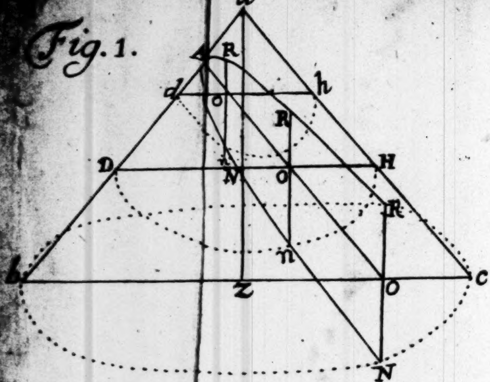
*Compend. Politicum*. An Account of the Troubles in the Regin of King *Henry* the 3d.

*Martindale*'s Book of Surveying.

Books of Riddles.

*FINIS.*

Fig. 1.

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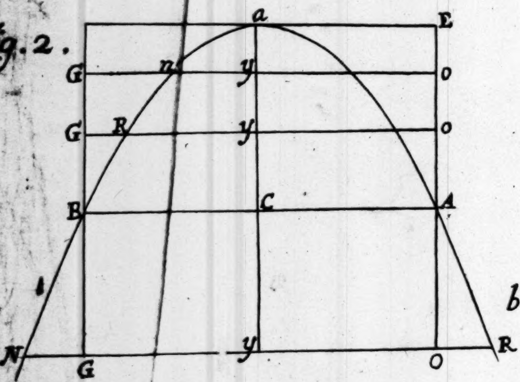


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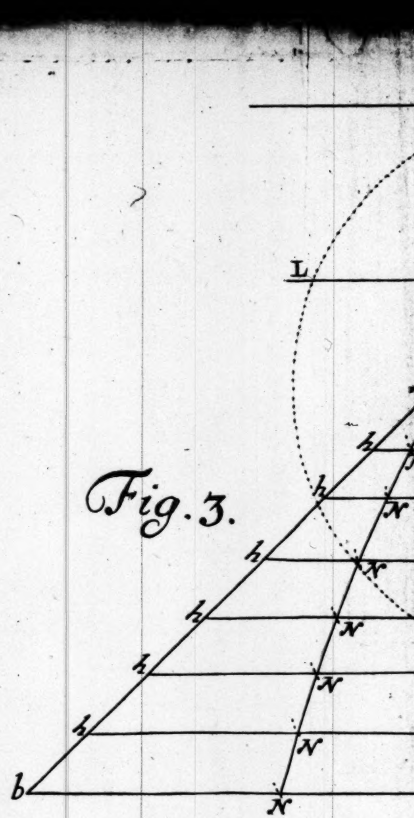


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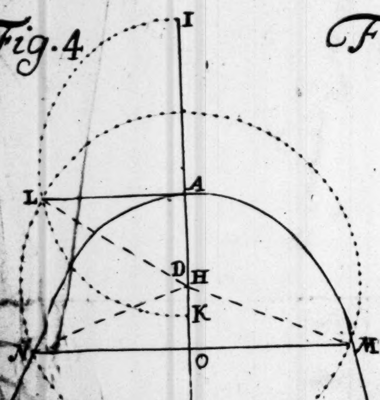


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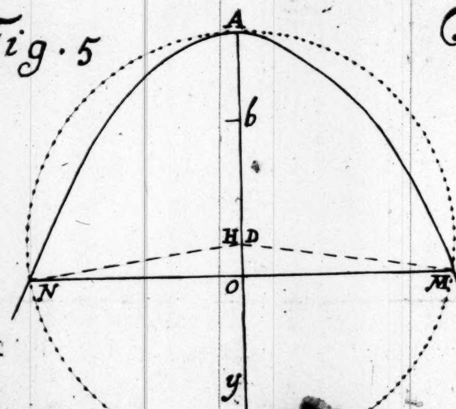


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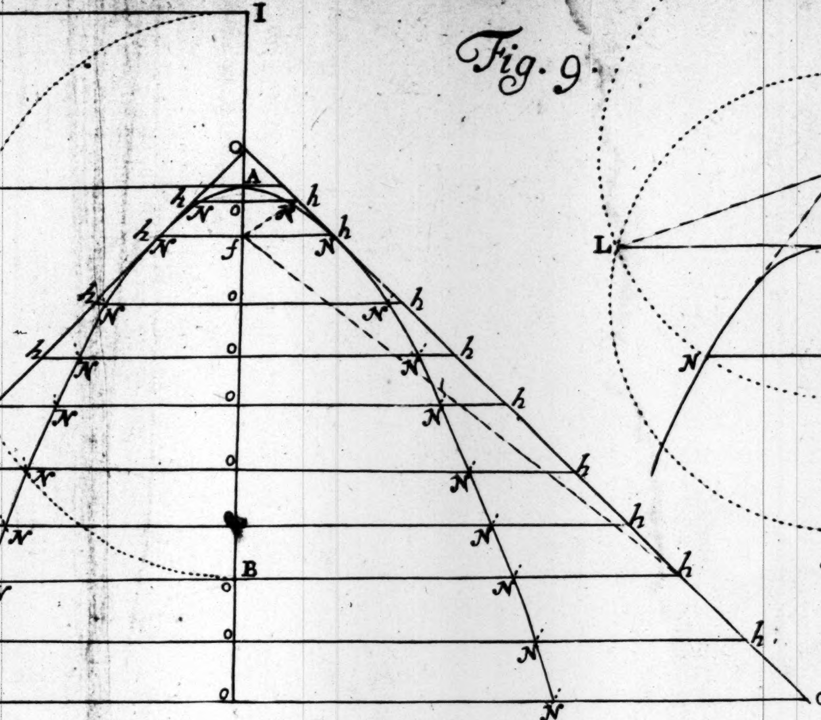


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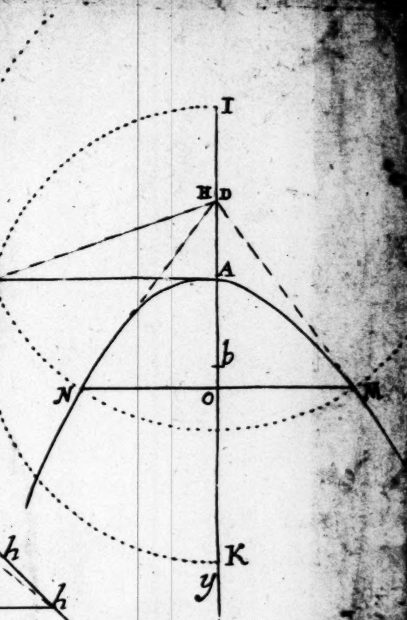


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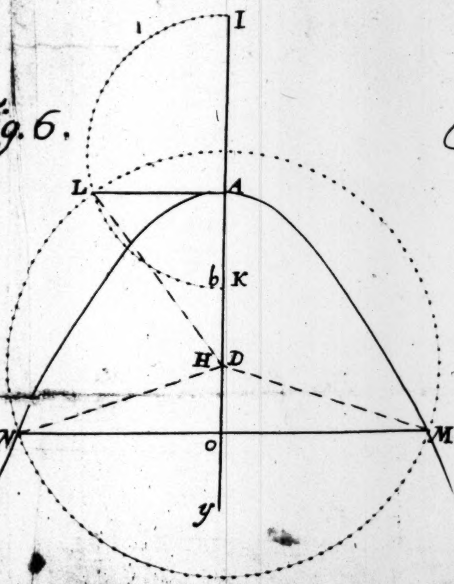



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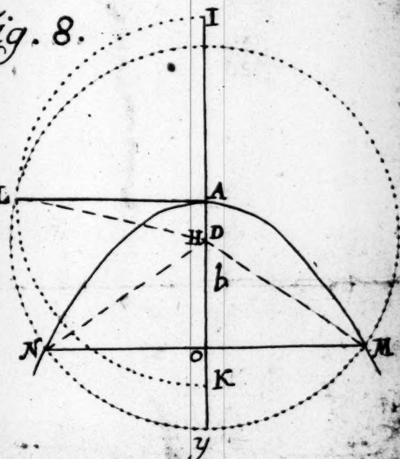

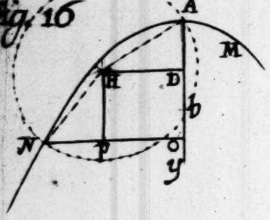
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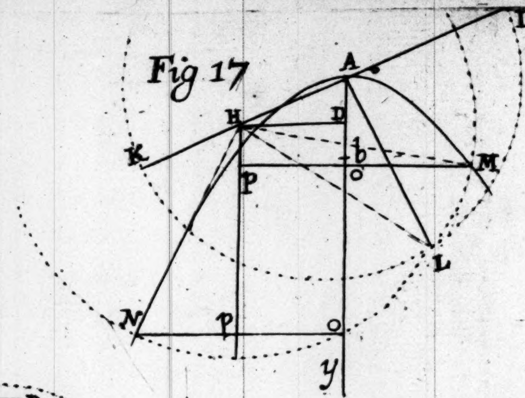
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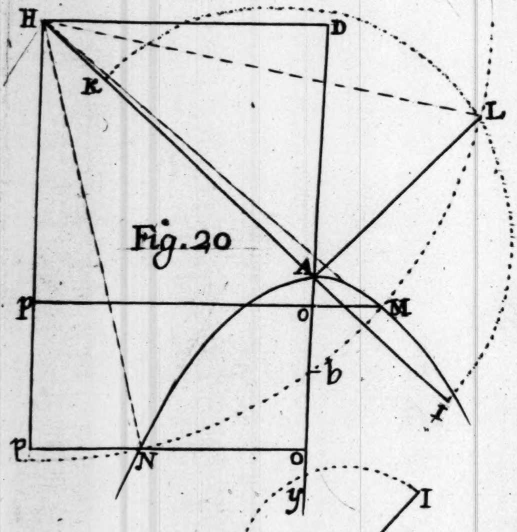
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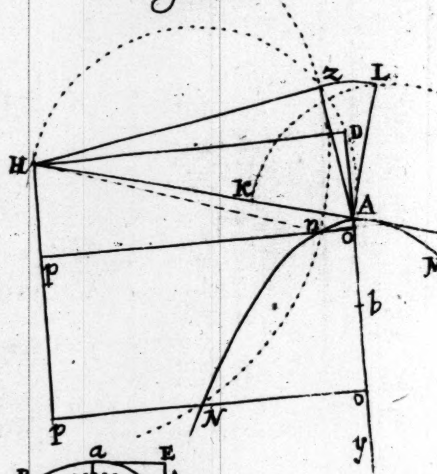
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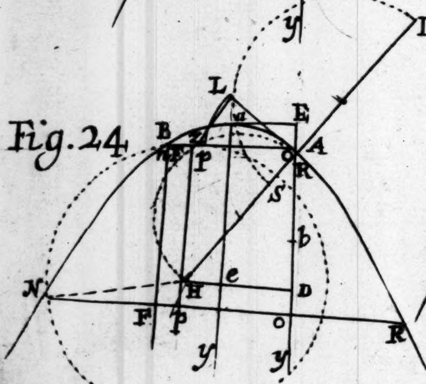
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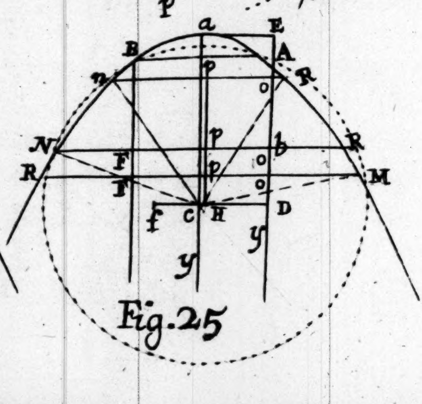
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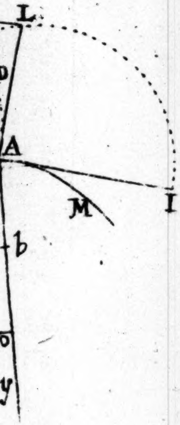


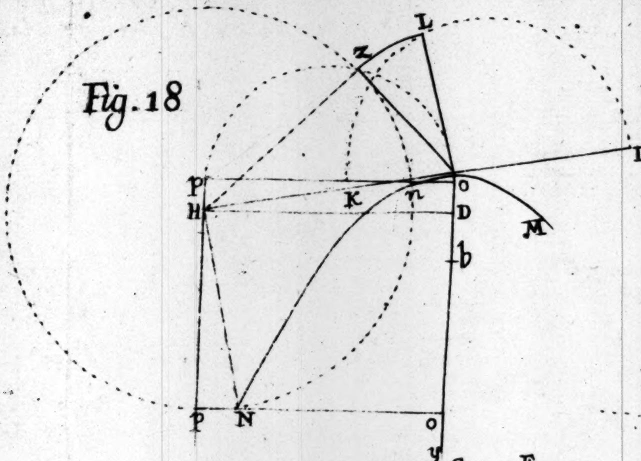

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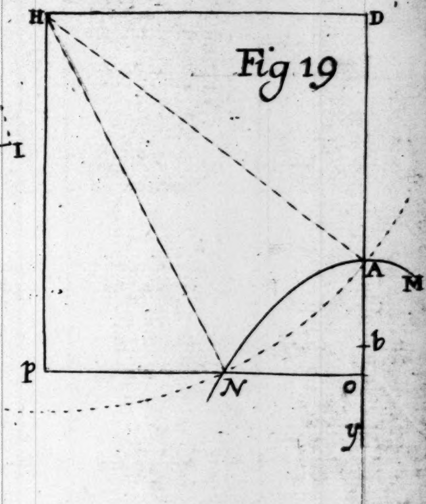

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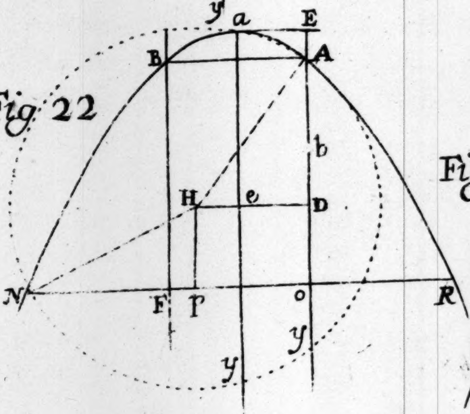

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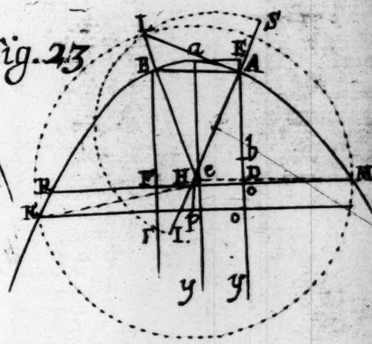

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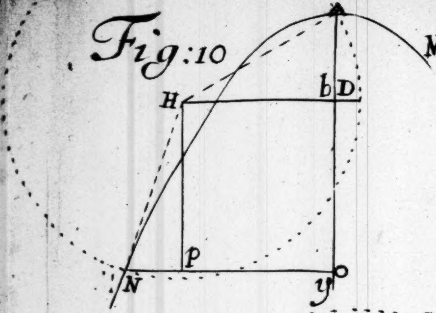
Fig:10

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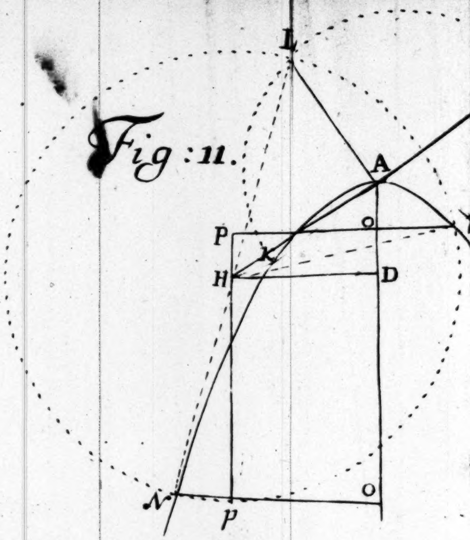


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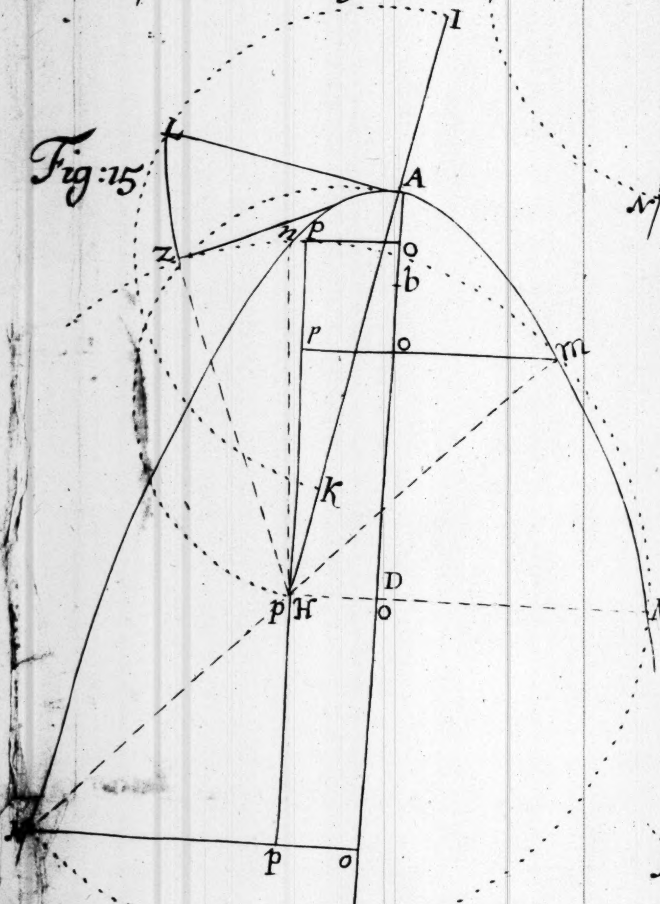


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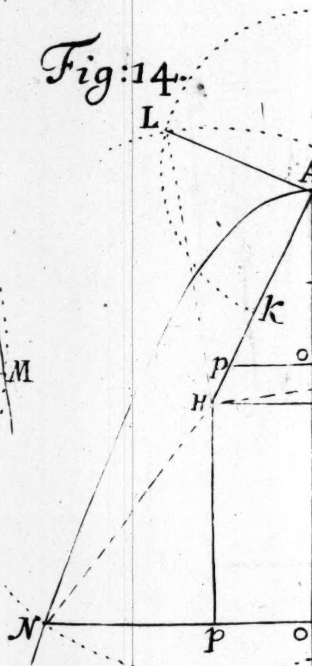

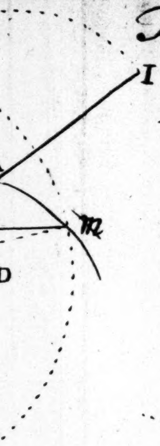


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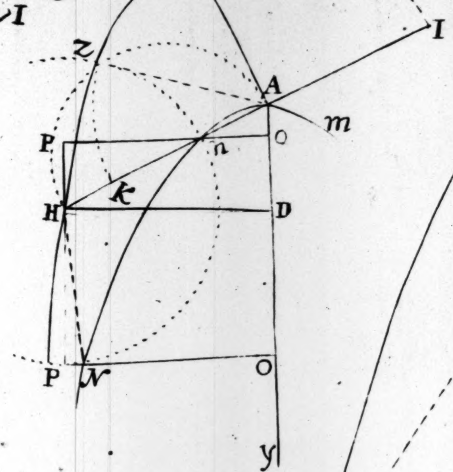


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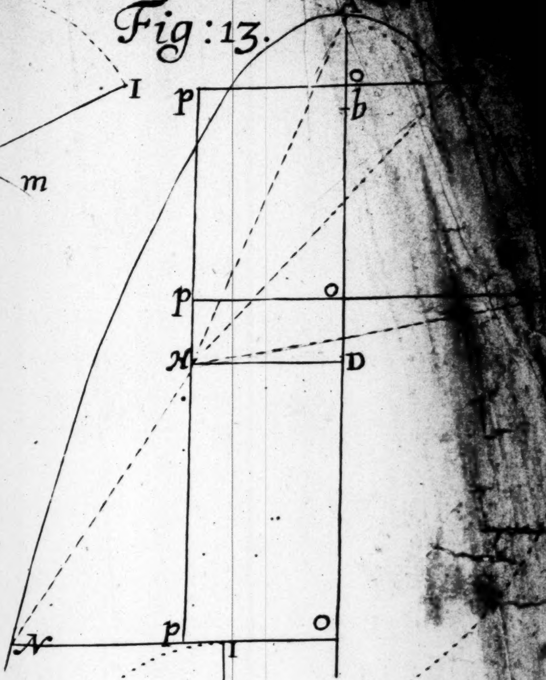


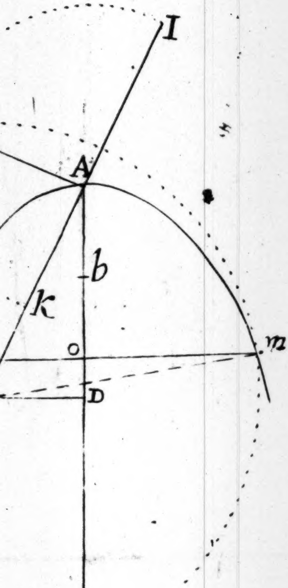


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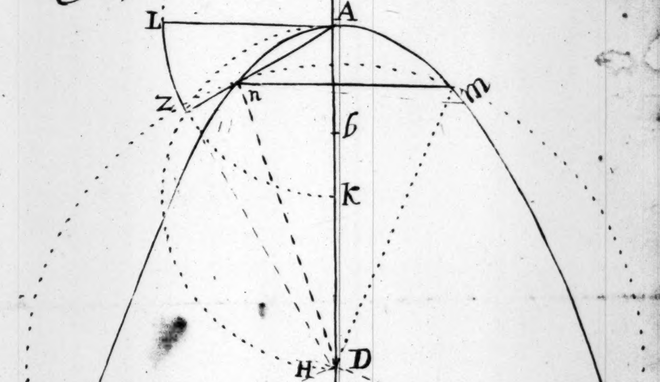

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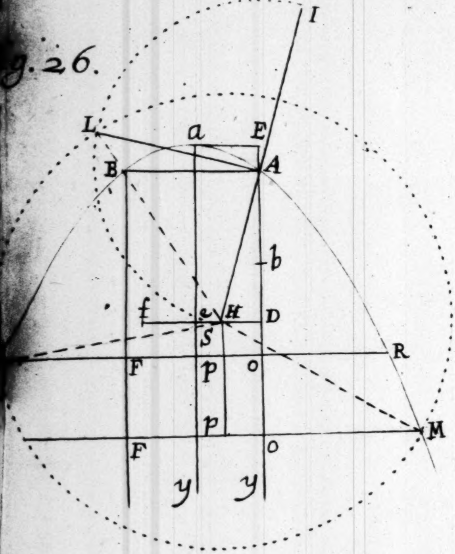

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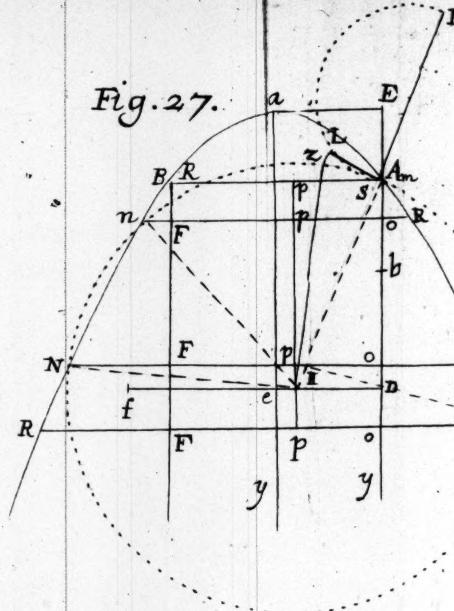

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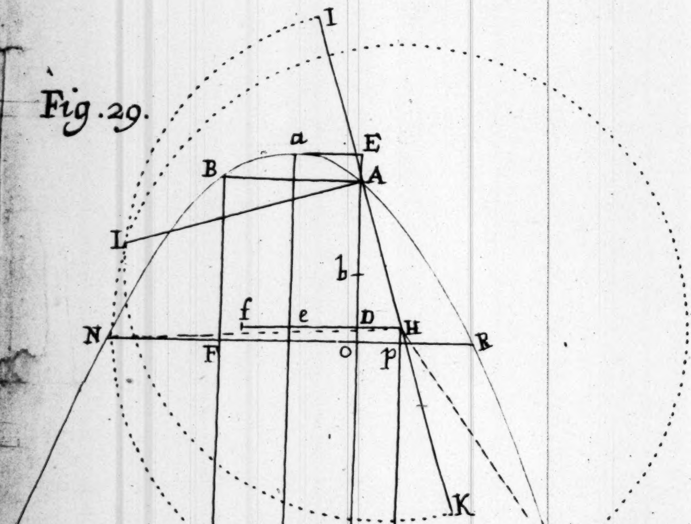

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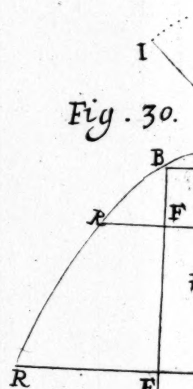

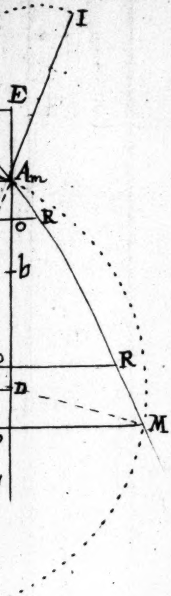


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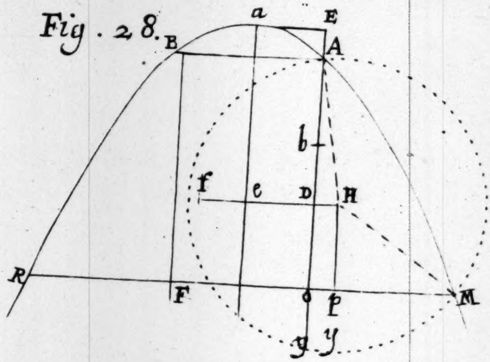


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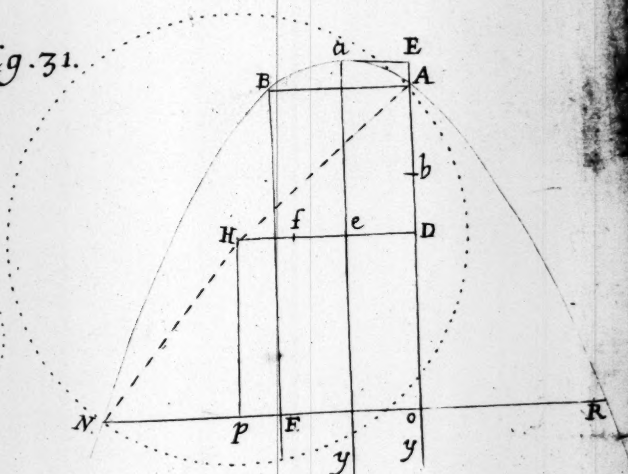


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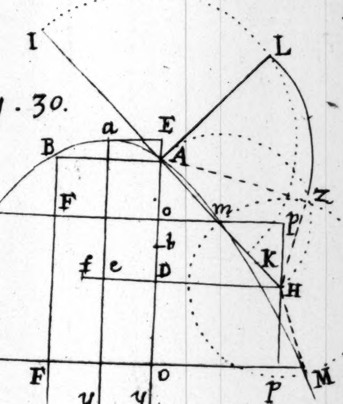

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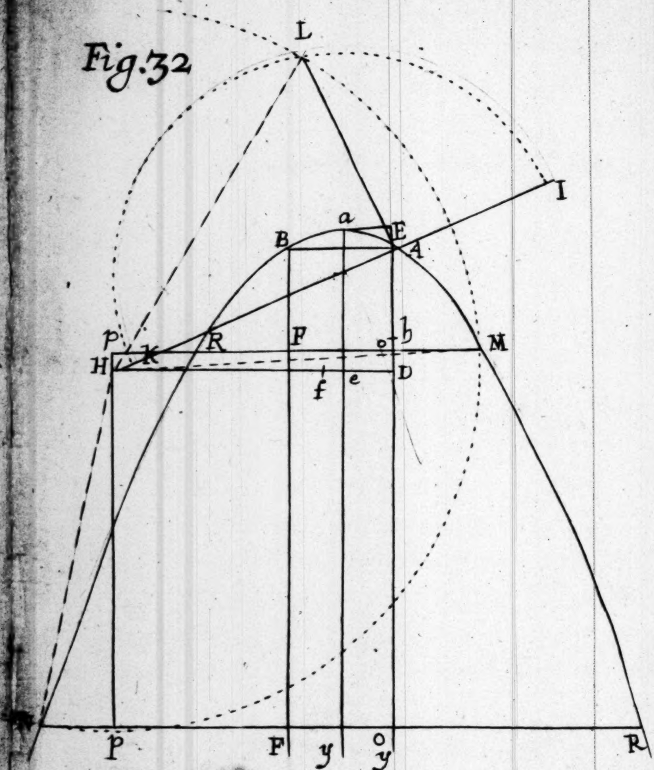


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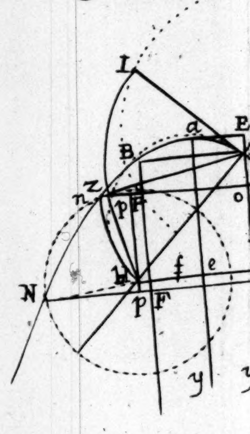


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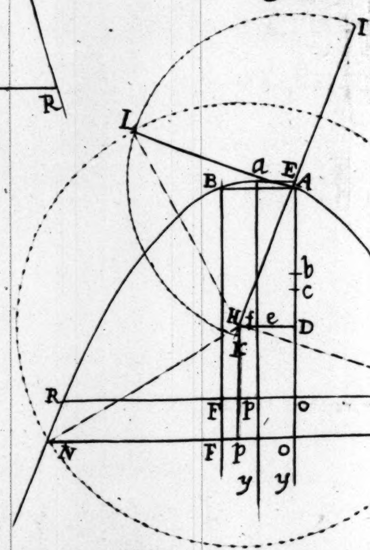


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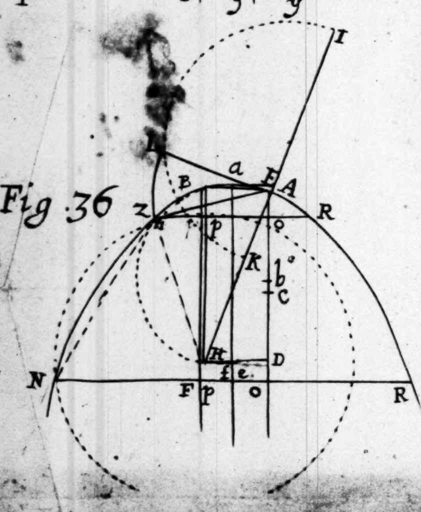

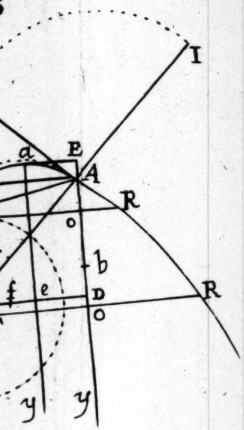


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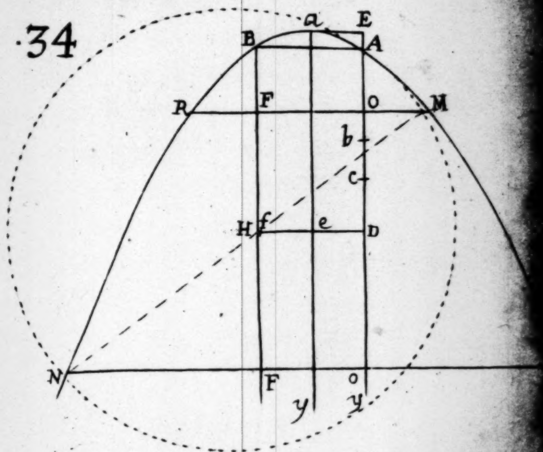


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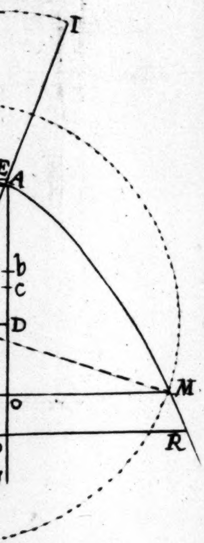


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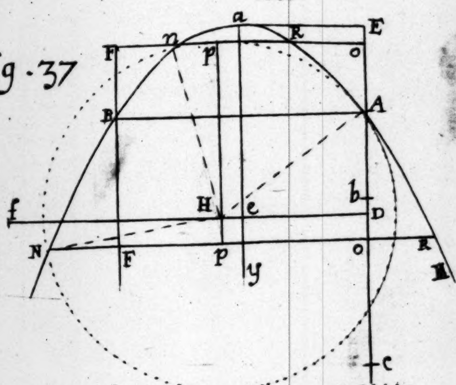


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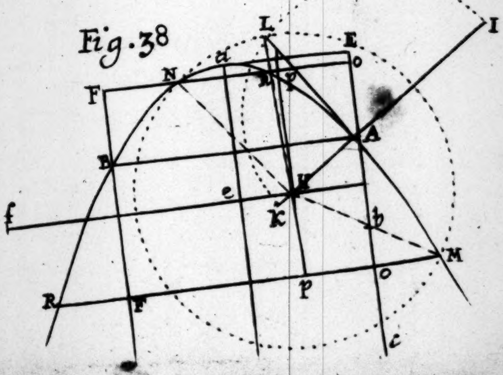

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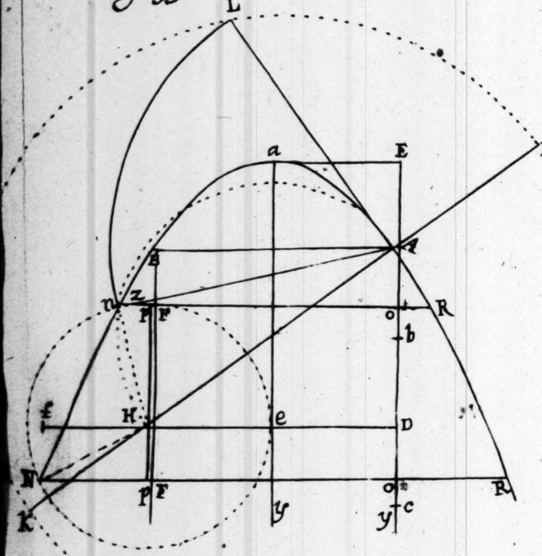

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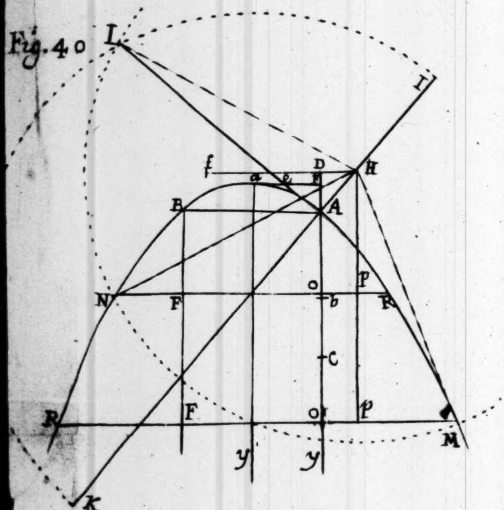

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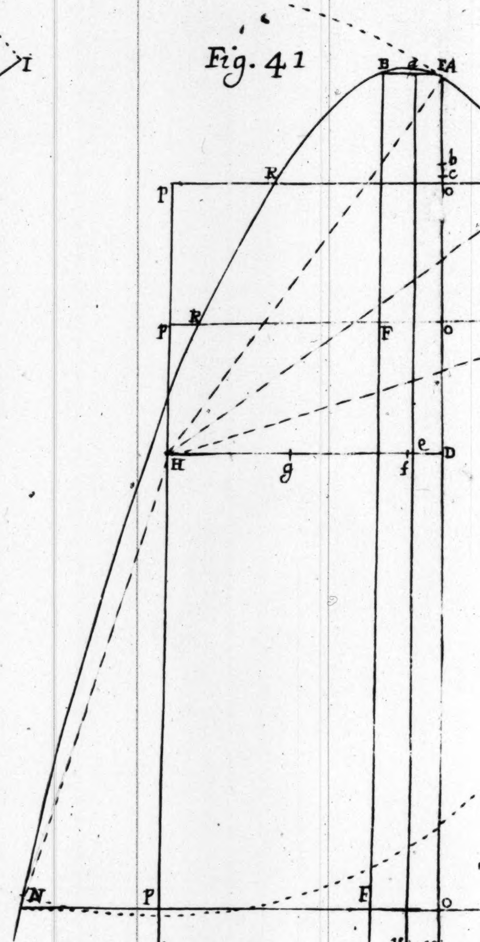


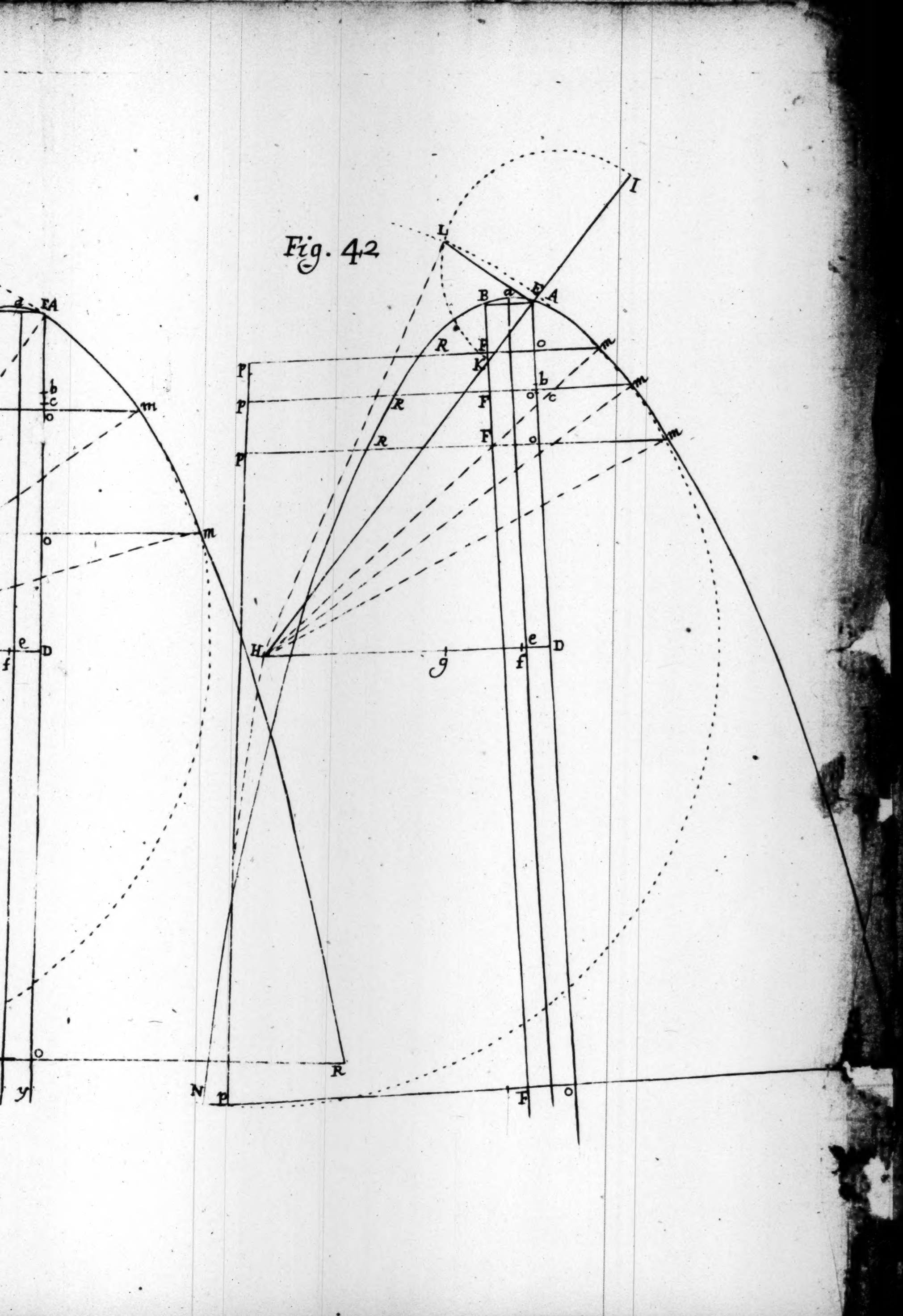
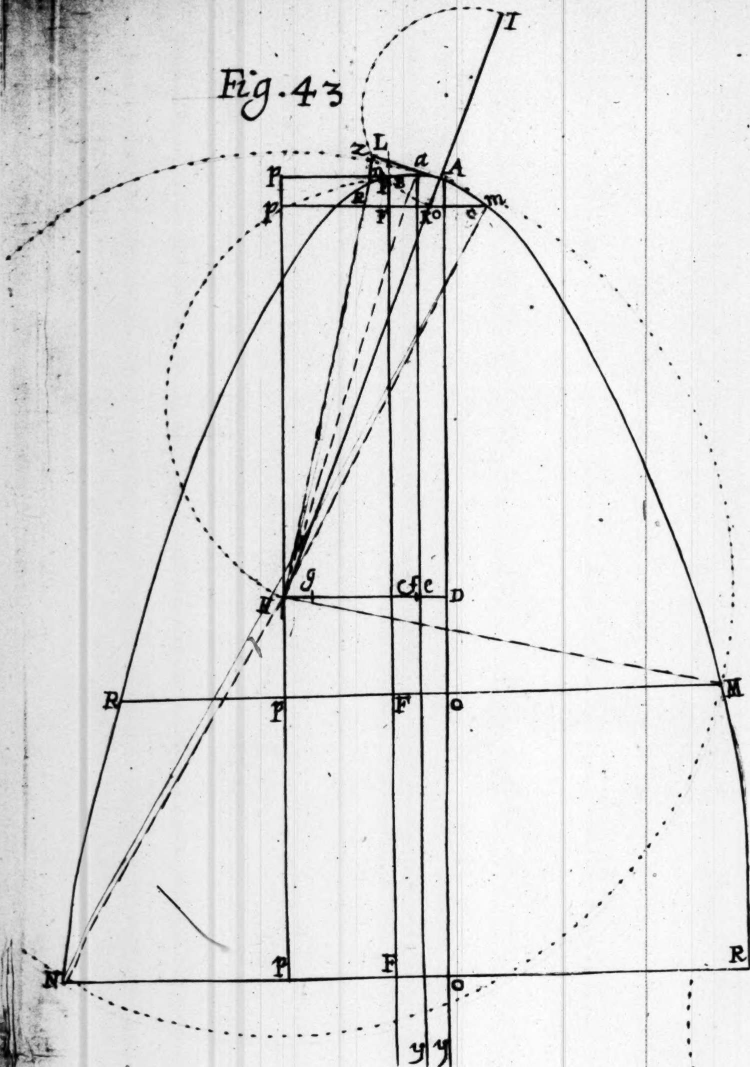


Fig. 42

Fig. 43

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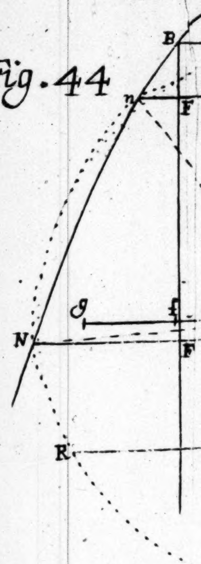


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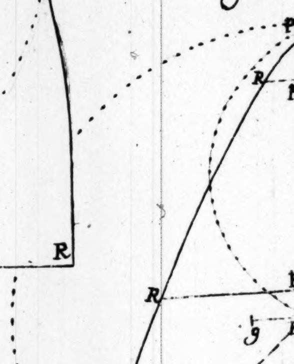

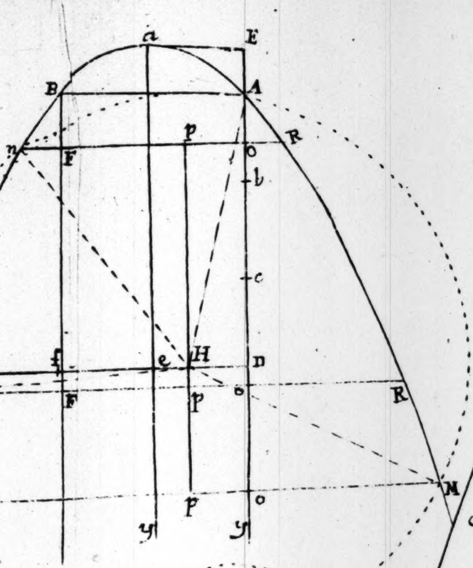

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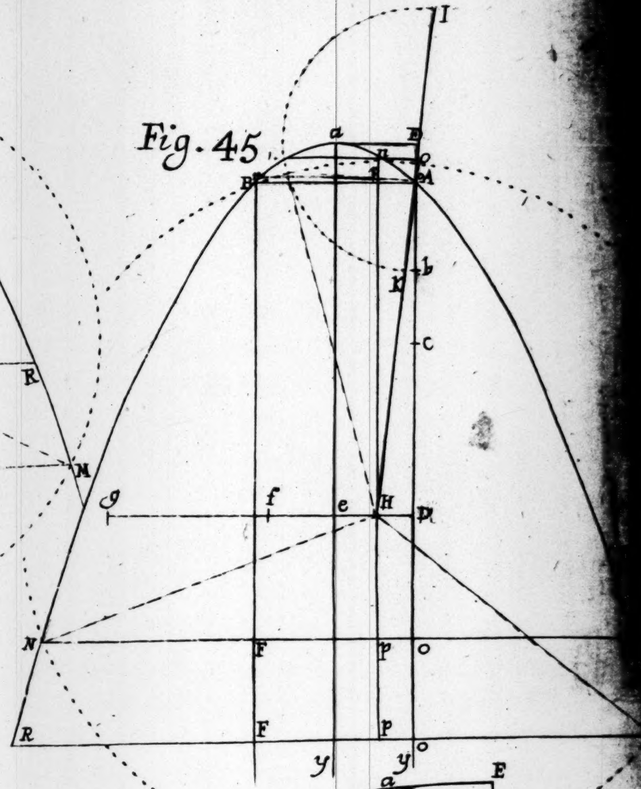

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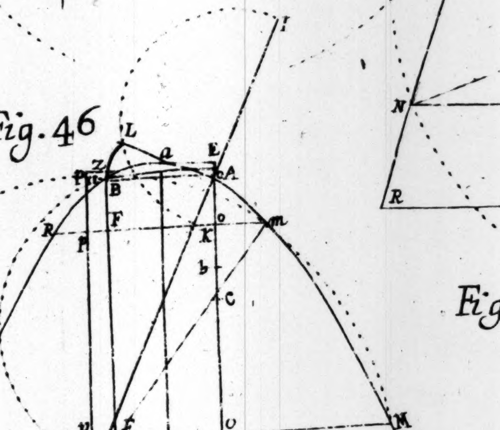

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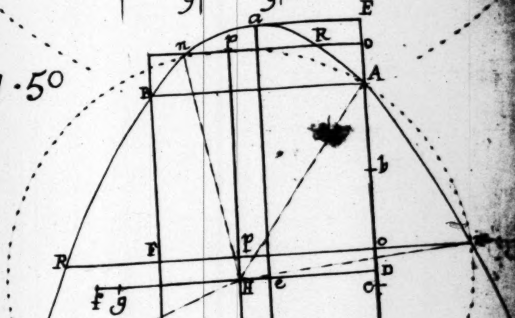

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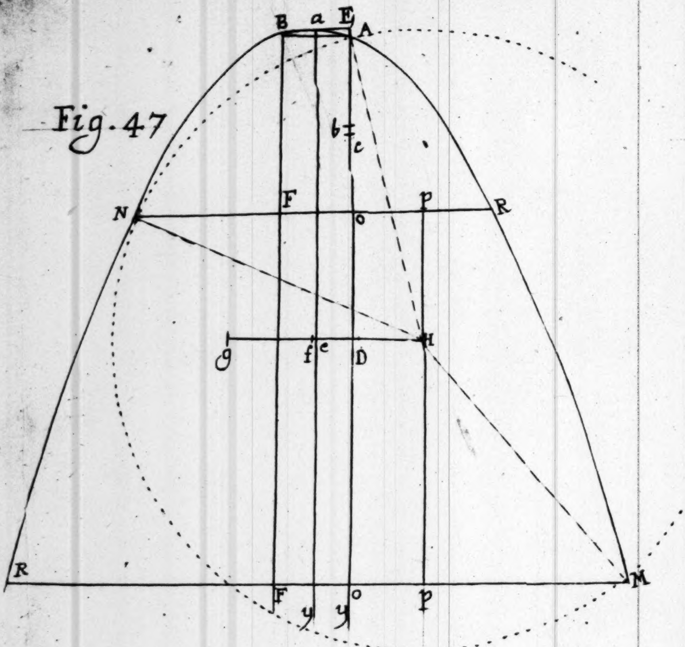


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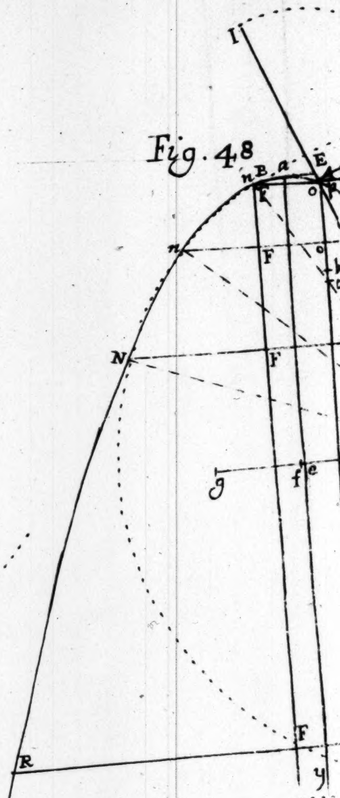


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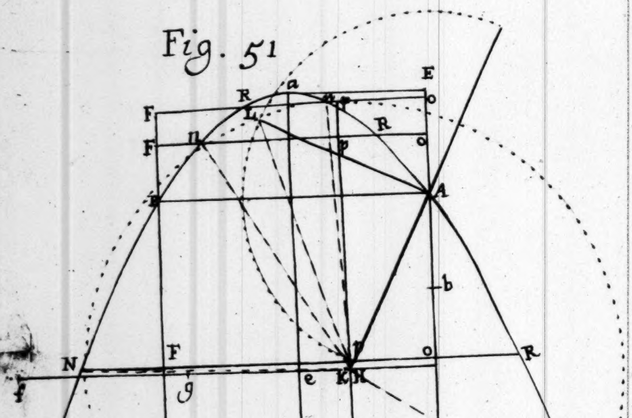


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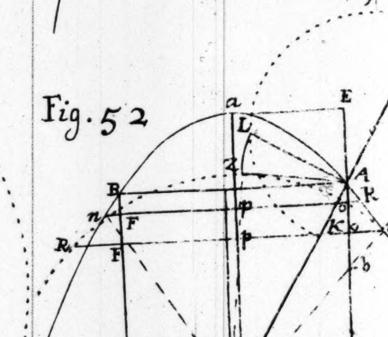

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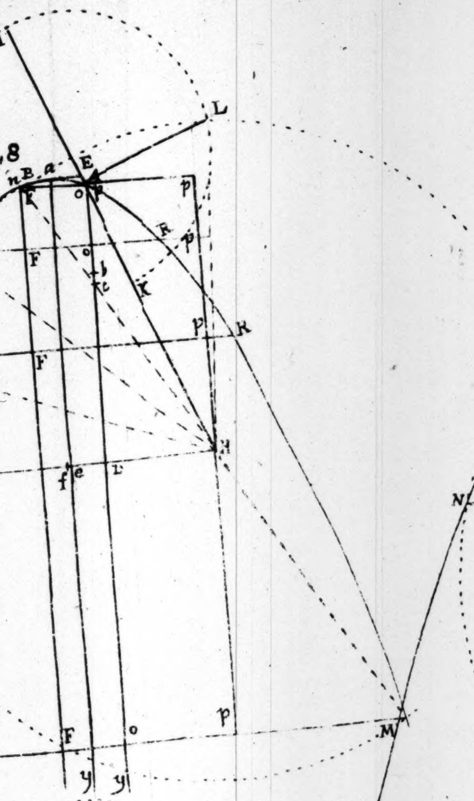


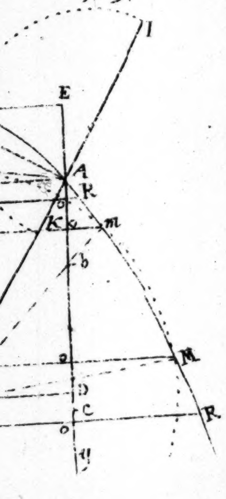


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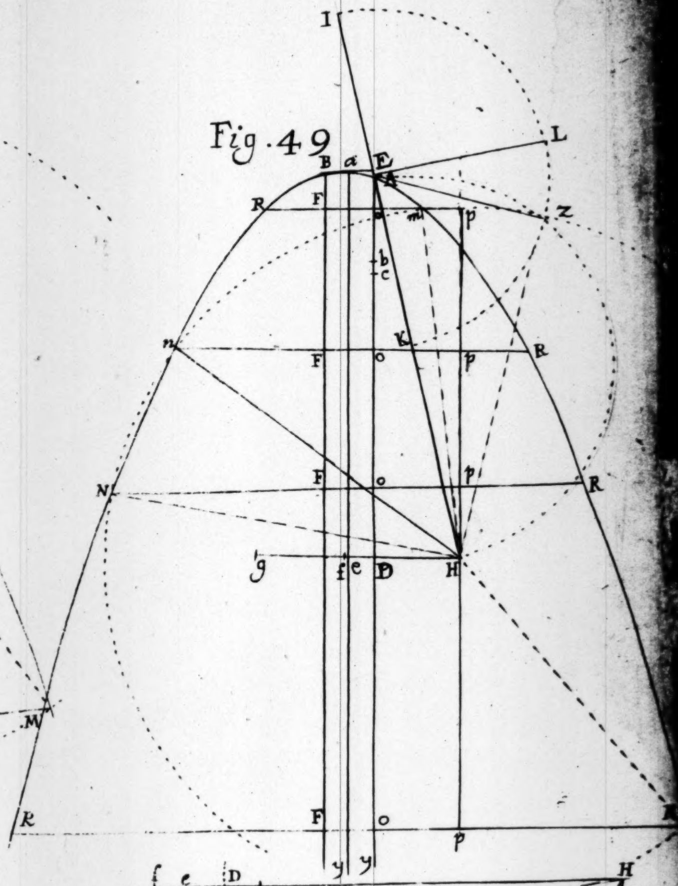


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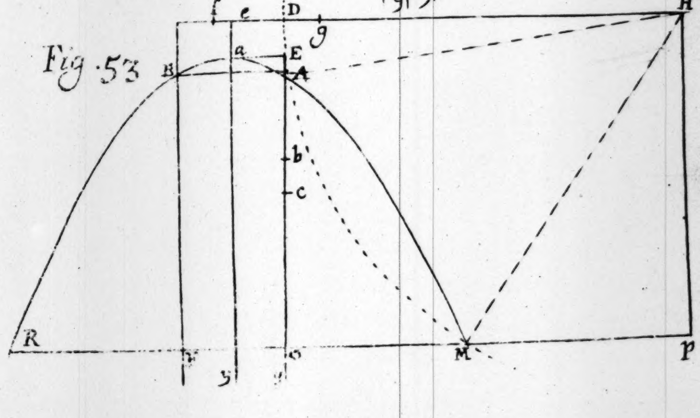

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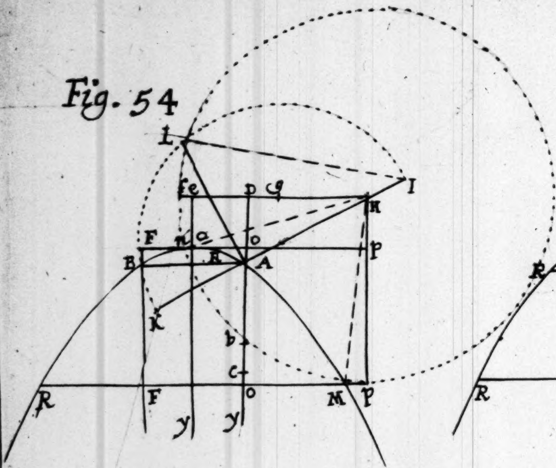

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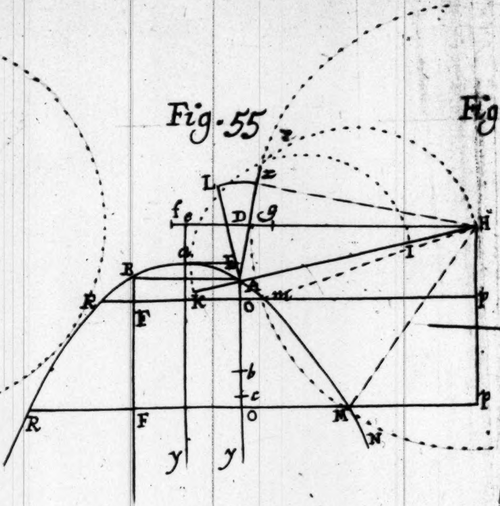
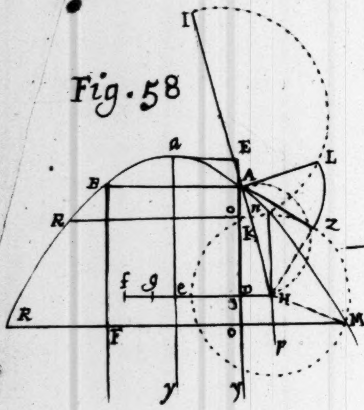


Fig.

Fig. 58



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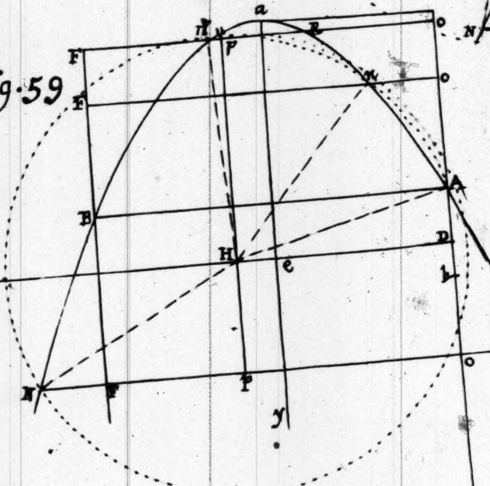

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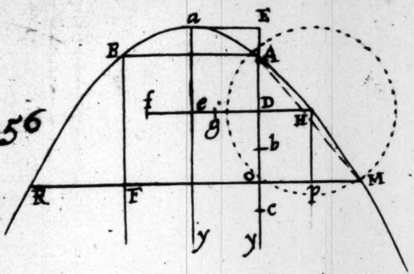

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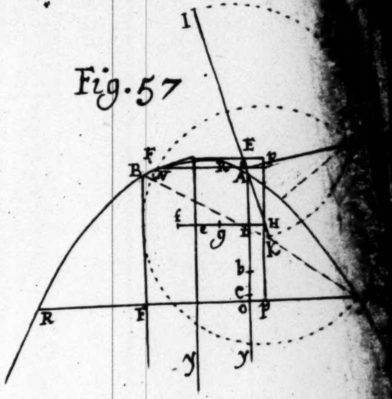
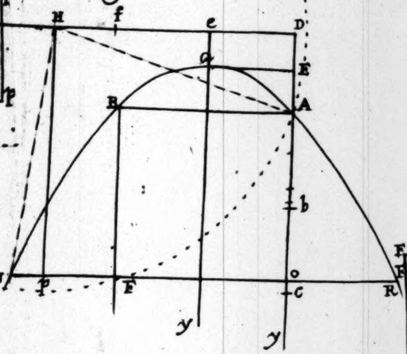


Fig. 62



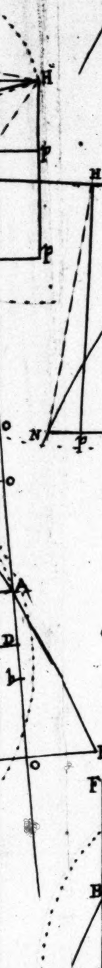


Fig. 60

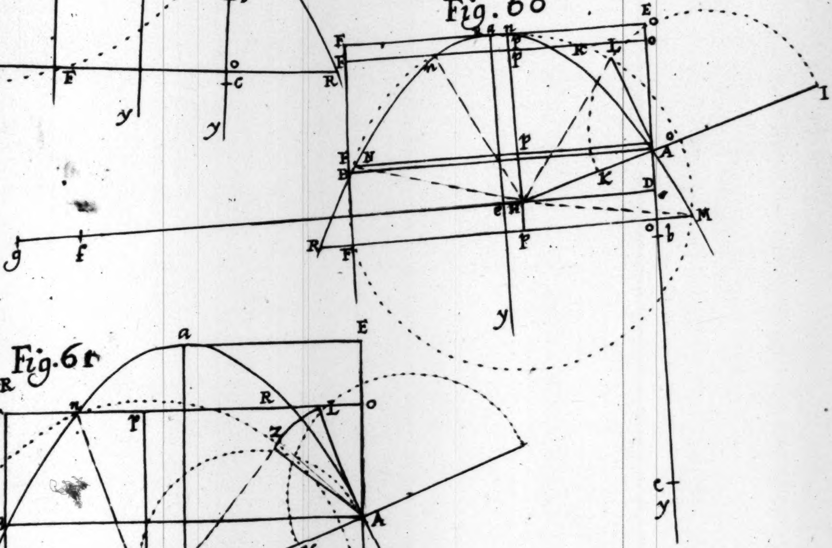


Fig. 61

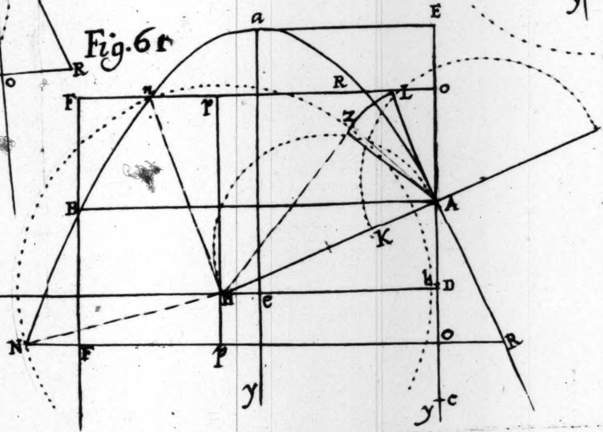


Fig. 63

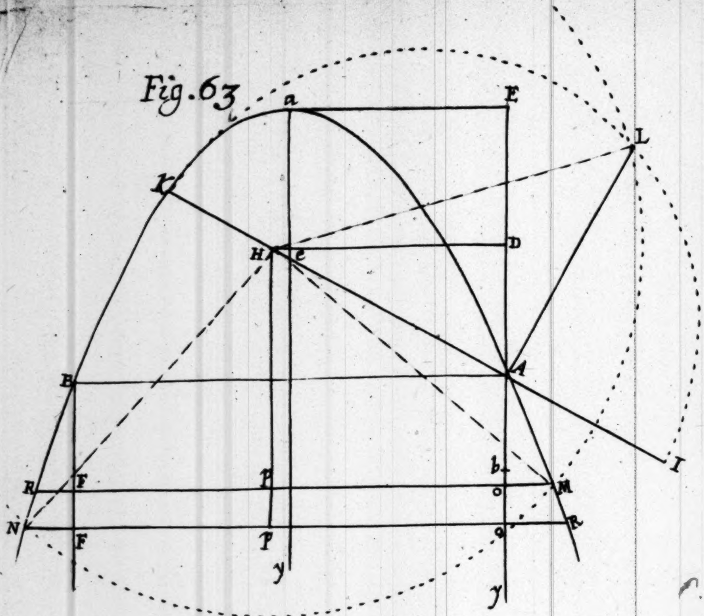



Fig. 64


Fig. 65

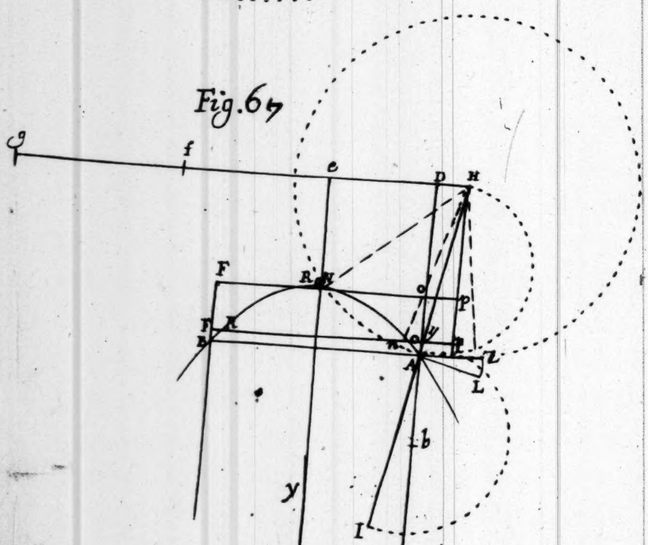
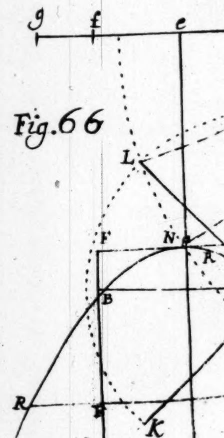



Fig. 66



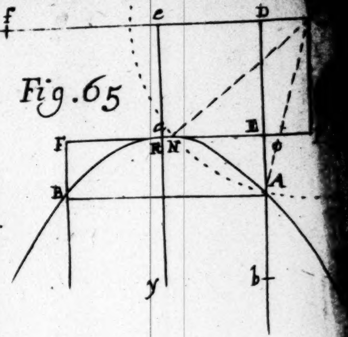


Fig. 65

Fig. 64